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The role of Barents–Kara sea ice loss in projected polar vortex changes

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Calculation of Bayes Factor

Our deductive approach (on the basis of Fig. 1, Fig. 2 and previous research) means that we do not seek evidence for our hypothesis of an influence of BK-SIC on SPV from Figure 6; instead, we seek to explore the implications of our hypothesis on the interpretation of the data. Nevertheless, it is appropriate to ask whether the data in Fig. 6 provide any evidence *against* our hypothesis; in other words, whether they should cause us to rethink our assumptions. Specifically, we ask whether the CMIP5 RCP8.5 projections of SPV, BK-SIC and T give reason to believe that the alternative hypothesis H1: “BK-SIC is no source of non-linearity of future SPV changes” is more likely than our assumption H0: “BK-SIC is a source of non-linearity, with a causal effect lying between 0.025 and 0.1”.

One can address this question by comparing the probability of seeing the data under H1 with that of the probability of seeing it under H0. This ratio is also known as the Bayes Factor (BF), and is used in Bayesian hypothesis testing (Jeffreys, 1961; Shikano, 2019; Wagenmakers, 2007):

$$\mathrm{BF} = \mathrm{P}(\mathrm{data} \mid \mathrm{H1}) \;/\; \mathrm{P}(\mathrm{data} \mid \mathrm{H0})\ .$$

A Bayes Factor much larger than unity implies the data to be much more likely under the alternative hypothesis and would thus motivate us to question our assumptions.

To calculate the BF, we first translate the two hypotheses into two linear regression models

$$\mathrm{H0{:}}\ \Delta\mathrm{SPV} = \mathrm{ce}\ \Delta\text{BK-SIC} + b_0\ \Delta\mathrm{T} + \varepsilon_0 \qquad \text{with ce in } [0.025, 0.1]$$

$$\mathrm{H1{:}}\ \Delta\mathrm{SPV} = b_1\ \Delta\mathrm{T} + \varepsilon_1$$

with ε_0, ε_1 denoting noise. Note that the Bayes Factor thus quantitatively describes whether the SPV changes as presented in Figure 6d are “more linear” than the Residuals in Fig. 6f, where the effect of sea ice has been removed in the latter.

We determine b_0 and b_1 for each climate model by regressing ΔSPV on ΔT (to estimate b_1) and by regressing (ΔSPV – ce ΔBK-SIC) on ΔT (to estimate b_0). We calculate different b_0 by iterating over different values of ce from 0.025 to 0.1 in steps of 0.005. We assume equal prior probability of all ce within the specified range, such that the denominator in the BF is simply the average over the different $\mathrm{P}(\mathrm{data} \mid \mathrm{H0}, \mathrm{ce}_i)_{i = 0.025, ..01}$. Since SPV exhibits multidecadal variability, but changes in SPV are calculated as the 30 year average relative to the reference period 1960-1989, we do not expect ΔSPV to pass through the origin and therefore also allow an intercept term when performing the regressions (which is however very small for most models).

For Gaussian noise ε_0 and ε_1, the probabilities P(data | H0) and P(data | H1) are also Gaussian and can thus be easily computed. More precisely, for normally distributed random noise ε_0 with zero mean and variance var(ε_0), the errors (i.e., projected minus predicted values of ΔSPV) follow the same distribution.

In order to calculate the BF it remains to calculate the variance of the noise terms. We estimate it here as the variance of SPV over the historical period (i.e., 1900-2005), further assuming that the effect of BK-SIC (as well as that of T) is negligible over his period, such that $var(\varepsilon_0) = var(\varepsilon_1)$. We then calculate the variance of the noise as the variance of the detrended "raw" (JFM mean) SPV data over the historical period, divided by 30, that is, the number of years used to calculate the smoothed ΔSPV data, consistent with a white noise assumption. Thus, we have:

$$\mathrm{var}(\varepsilon_0) = \mathrm{var}(\varepsilon_1) = \mathrm{var}(\mathrm{SPV}_{\mathrm{raw,\,hist}})/30.$$

Since the probability of observing different, independent data is the same as the product of the individual probabilities, and since the factors $\sqrt{2\pi\ \mathrm{var}(\varepsilon_0)}^{-1}$ and $\sqrt{2\pi\ \mathrm{var}(\varepsilon_1)}^{-1}$ in the probability density functions are the same for H0 and H1, it follows that:

$$\mathrm{BF} = \mathrm{P}(\mathrm{data} \mid \mathrm{H1}) \,/\, \mathrm{P}(\mathrm{data} \mid \mathrm{H0})$$

$$= \mathrm{P}(\mathrm{data} \mid \mathrm{H1}) \,/\, \left(\frac{1}{\mathrm{n}} \sum_{1}^{\mathrm{n}} \mathrm{P}(\mathrm{data} \mid \mathrm{H0}, \mathrm{ce_i})\right)$$

$$= \exp\left(\frac{-\mathrm{SSE}_1}{2\,\mathrm{var}(\varepsilon_1)}\right) \Big/ \left(\frac{1}{\mathrm{n}} \sum_{\mathrm{i}}^{\mathrm{n}} \exp\left(\frac{-\mathrm{SSE}_{0,\,\mathrm{ce_i}}}{2\,\mathrm{var}(\varepsilon_0)}\right)\right)$$

where SSE_1 and SSE_0 denote the sum of squared errors under H1 and H0 and the index i runs over the n different ce between 0.025 and 0.1 (in steps of 0.005).

The results from the regressions are shown in Fig. S1 for all models, with results for H1 indicated in grey, and for H0 (for the most extreme case of ce = 0.1) in blue. The Bayes Factors are indicated in each plot. Note that it is common practice in Bayesian hypothesis testing to only consider Bayes Factors exceeding 20 as strong evidence against H0 with, e.g. a BF between 1 and 3 seen as "not worth more than a bare mention" (Jeffreys, 1961). Here, the key result is that for almost all CMIP5 models, the Bayes Factor is close to unity, meaning that the data are similarly likely under both hypotheses. Only for model 11 (BF = 20.7), model 14 (BF = 4.7) and model 23 (BF = 4.4) does the data appear more likely under H1. However, for these models the errors for H0 are actually quite small, making H0 not unreasonable, given the involved simplifications.

In summary, the CMIP5 projections shown in Fig. 6 do not provide evidence against our H0, and therefore justify keeping our deductive approach on the basis of Fig. 1 and Fig. 2.

References

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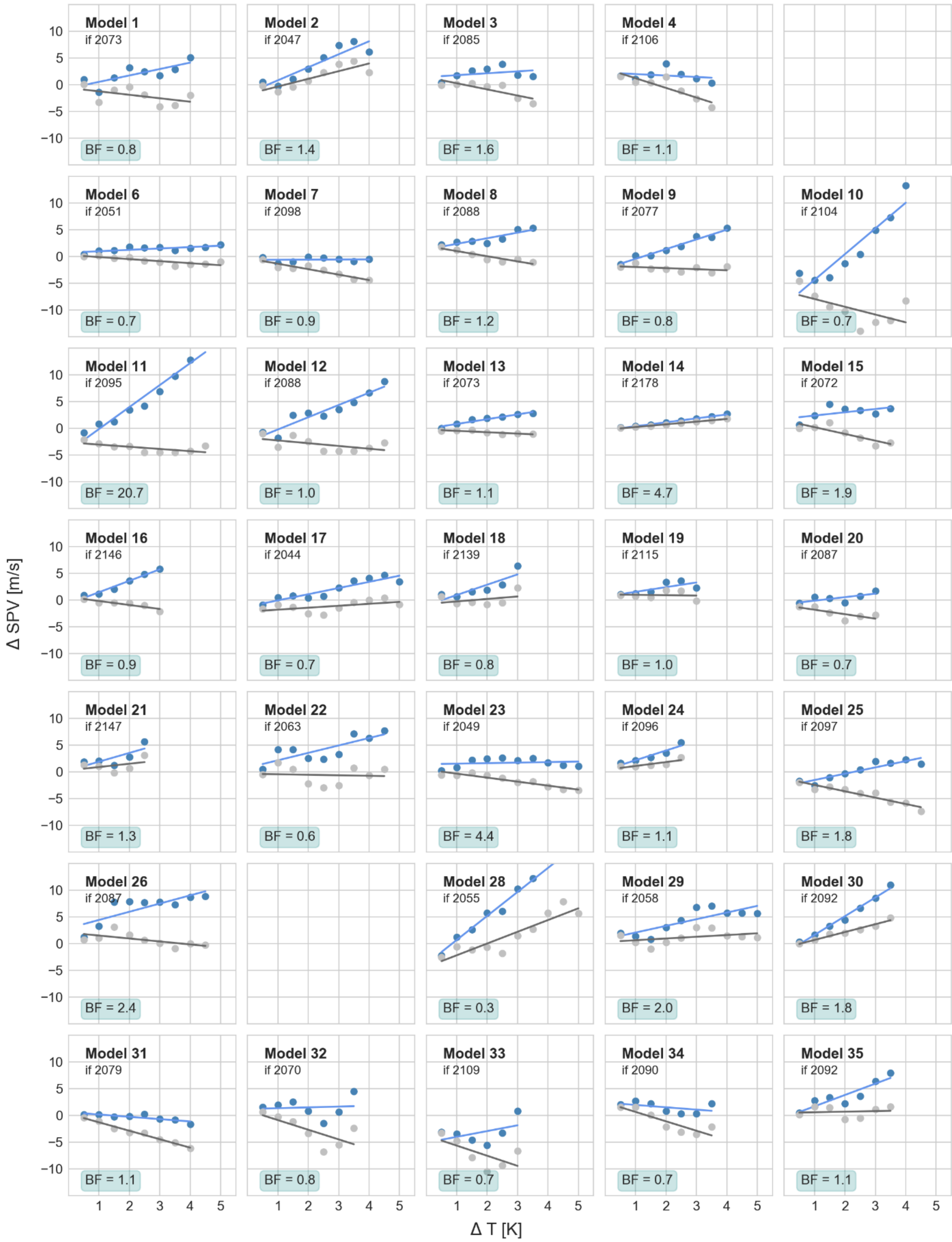

Fig. S1. Bayes factor analysis.

Shown are, for each CMIP5 model, ΔSPV as a function of ΔT (grey dots, as in Fig. 6d) as well as the Residuals, i.e. (ΔSPV – ce BK-SIC), for the most extreme case of ce = 0.1 (blue dots; similar to Fig. 6f, but note that the Residuals in Fig. 6f are shown for ce = 0.05). The lines show b_0 ΔT (in blue) and b_1 ΔT (in grey). The Bayes Factor (BF) of each model as well as the year the BK Seas become ice-free (if) are indicated in each panel.