Supplement of Weather Clim. Dynam., 4, 1087–1109, 2023
https://doi.org/10.5194/wcd-4-1087-2023-supplement

Supplement of

Identifying quasi-periodic variability using multivariate empirical mode decomposition: a case of the tropical Pacific

Lina Boljka et al.

Correspondence to: Lina Boljka (lina.boljka@uib.no) and Nour-Eddine Omrani (noureddine.omrani@uib.no)

S.1 Supplementary Figures

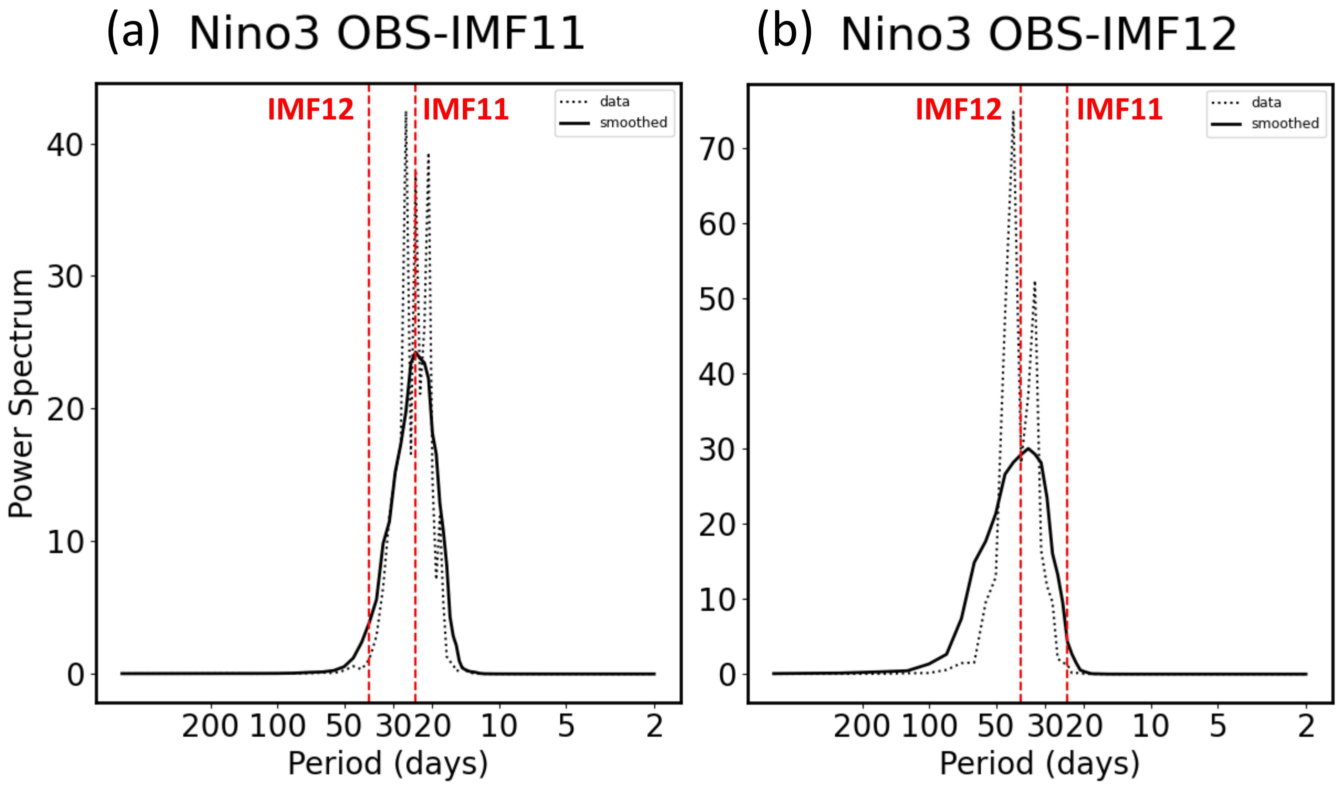

Figure S1. Power spectra of (a) IMF11 and (b) IMF12, their eastern Pacific SST (Niño3) index. Black dotted lines represent raw power spectra of IMFs, black solid line is 10-point smoothing of the raw power spectra, and red dashed lines represent average frequencies of IMF11 and IMF12 (as labelled) — for values see the main text or Table S1 (second column).

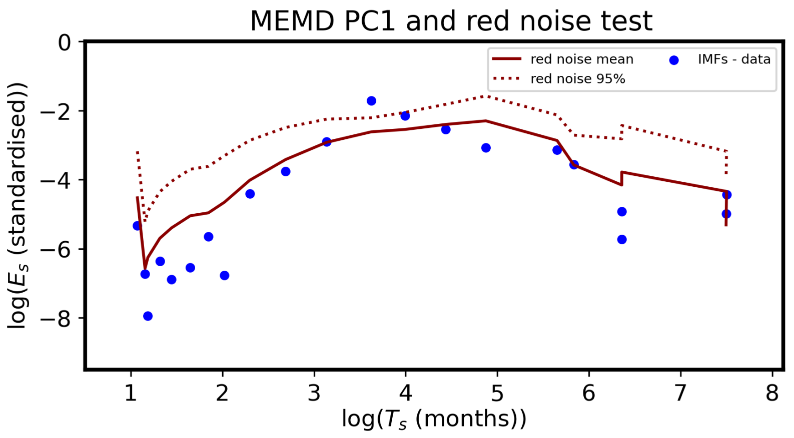

Figure S2. As Fig. 2 in the main text but for IMFs of PC1 of the combined field (via MEMD; blue dots) instead of eastern Pacific SST (Niño3) index.

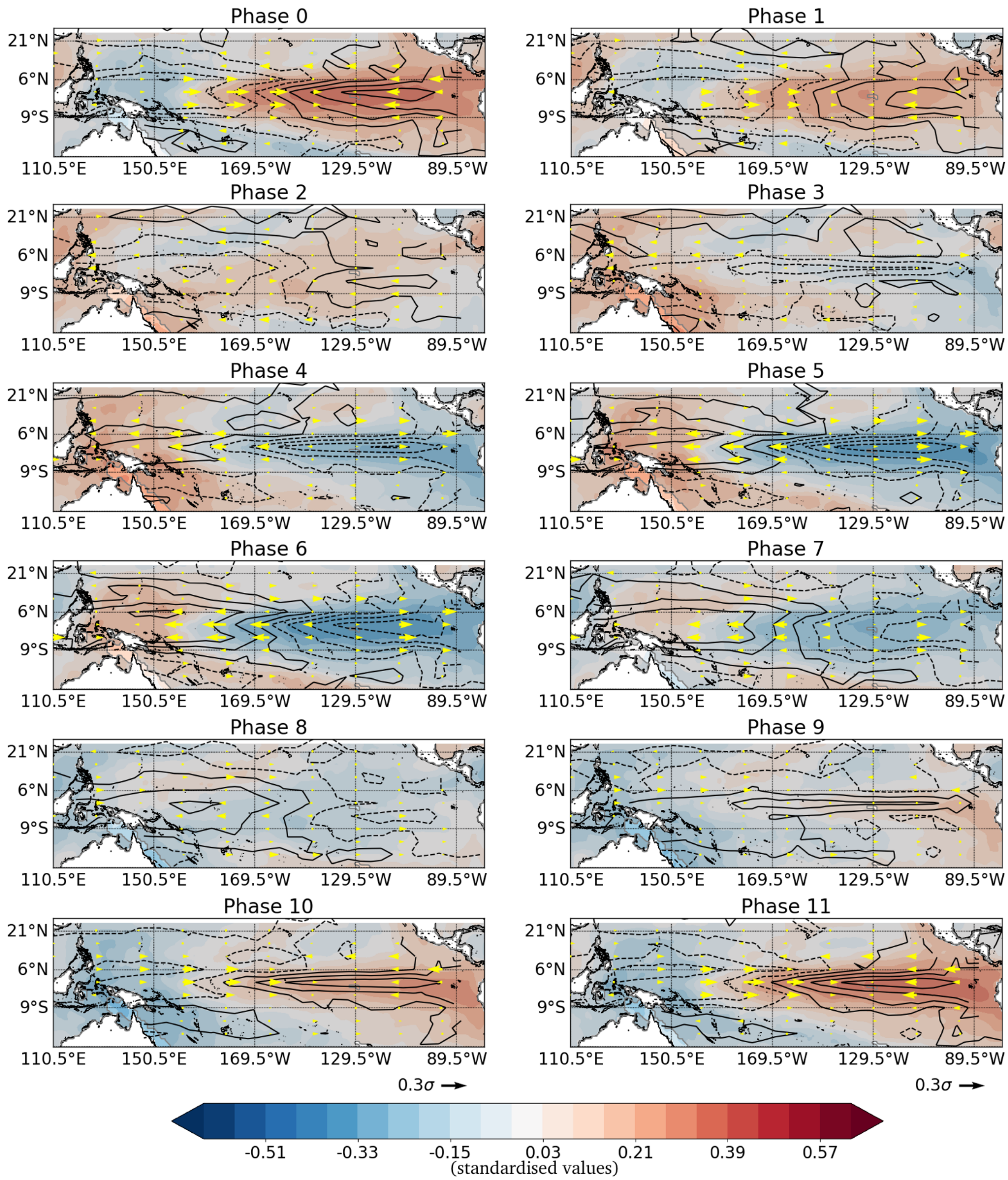

Figure S3. As Fig. 5 in the main text but for IMF11.

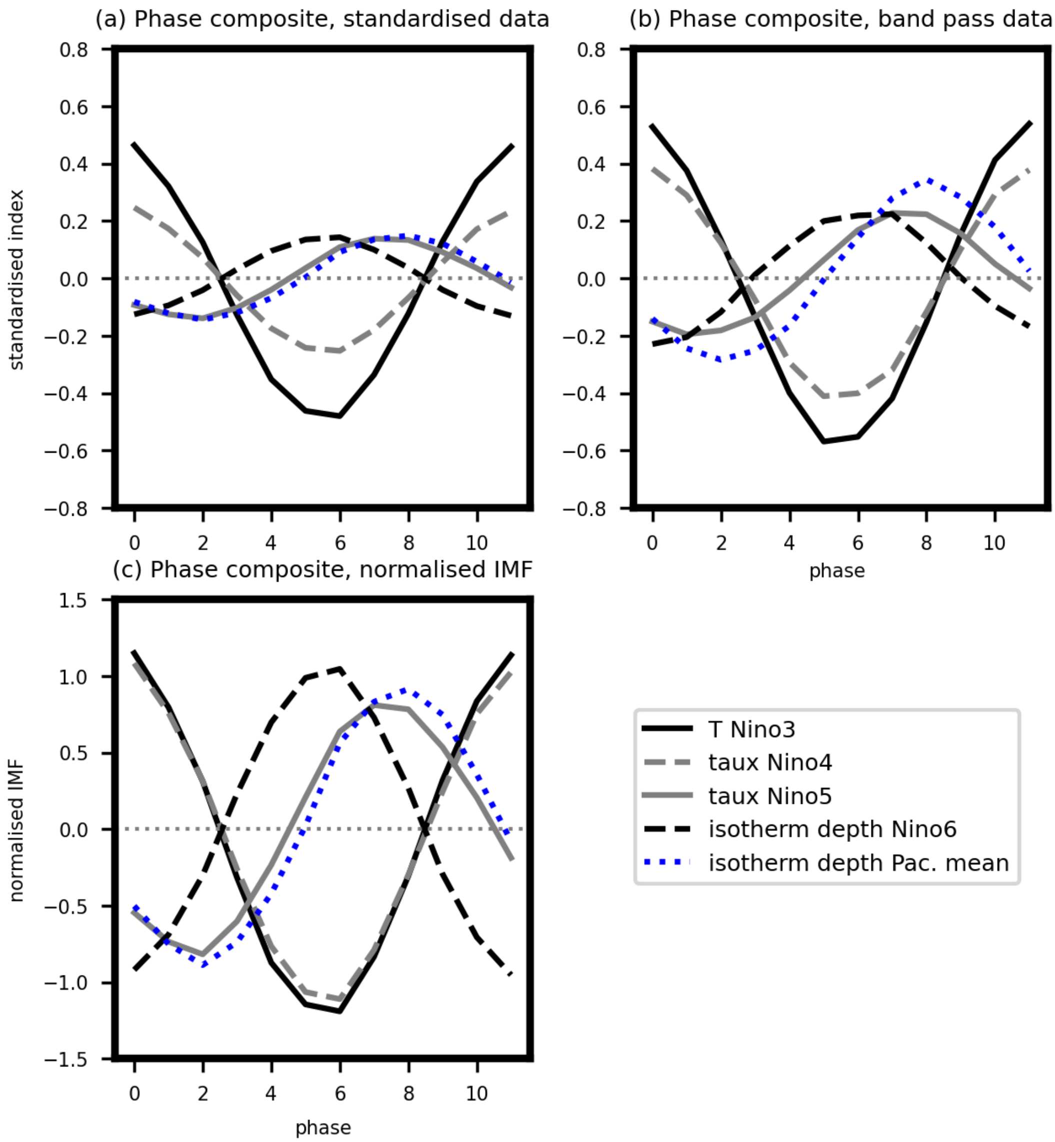

Figure S4. As Fig. 6 in the main text but for IMF11.

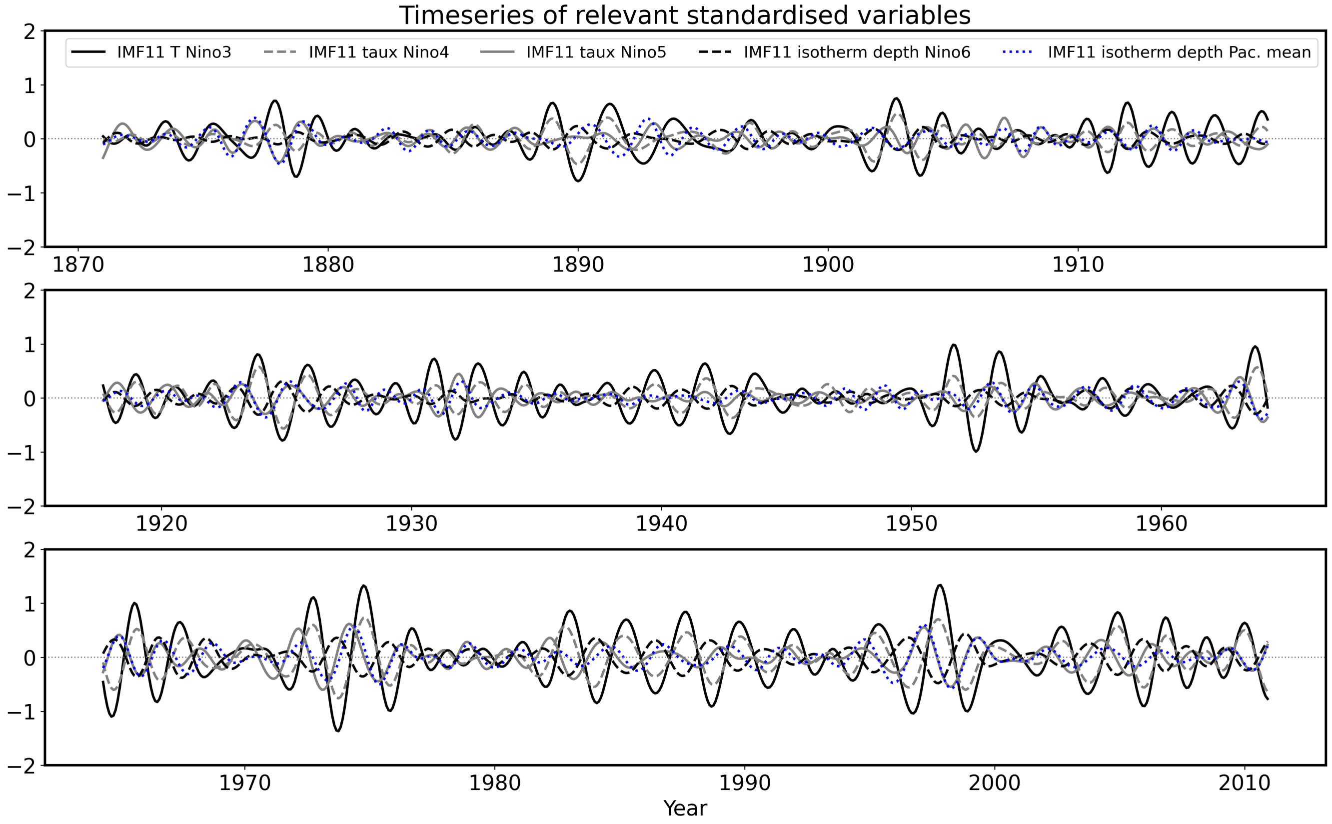

Figure S5. As Fig. 7 in the main text but for IMF11.

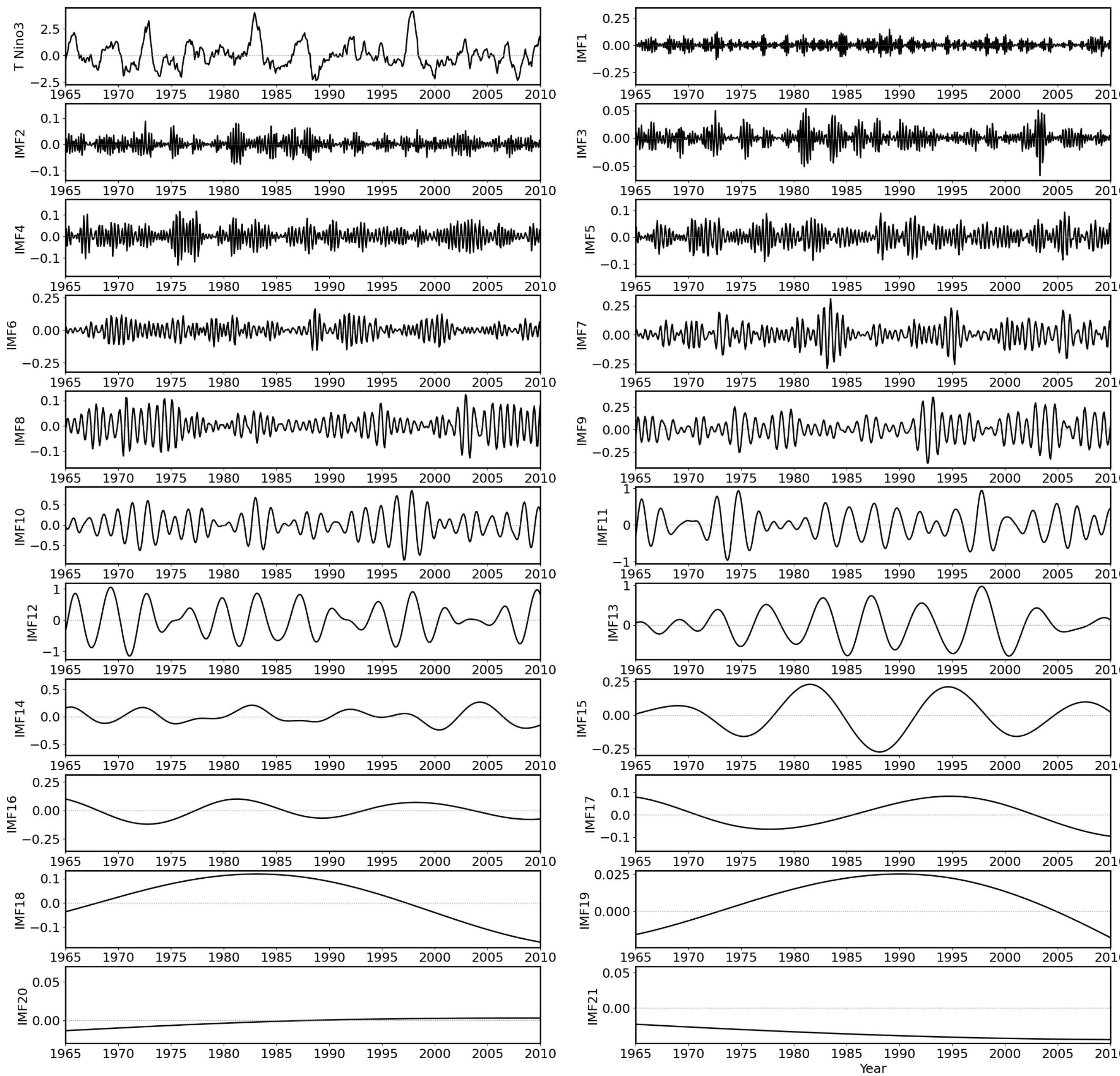

Figure S6. Timeseries of eastern Pacific SST (Niño3) from input data (top left panel) and IMFs as inferred via MEMD analysis for the same variable (see other panels as labelled). For clarity only values between 1965 and 2010 are shown. Note that amplitudes of different modes vary, i.e., y-axis is not the same in all panels. For characteristic periods of IMFs see Table S1 (second column).

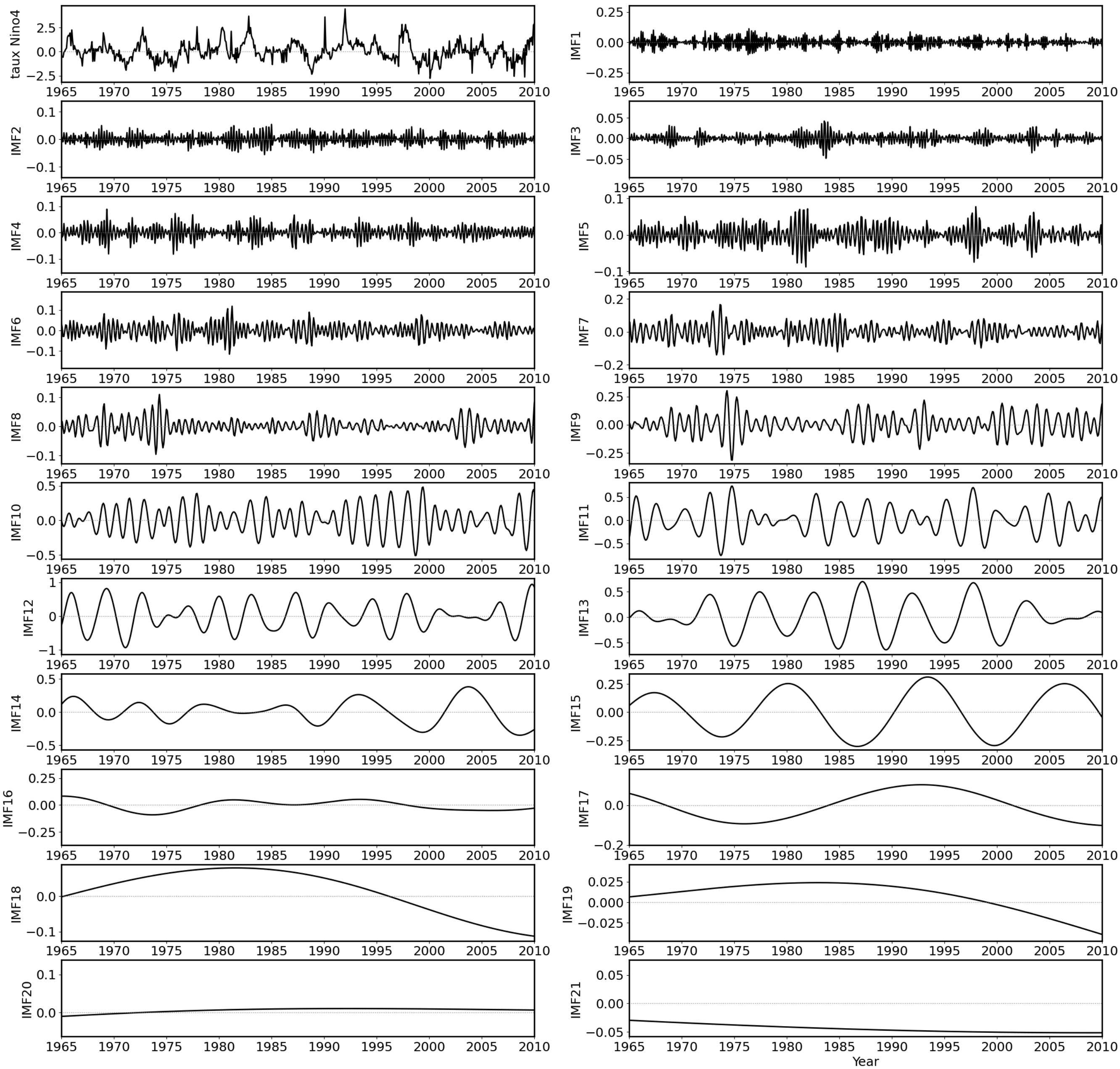

Figure S7. As Fig. S6 but for central Pacific τ_x (Niño4). For characteristic periods of IMFs see Table S1 (third column).

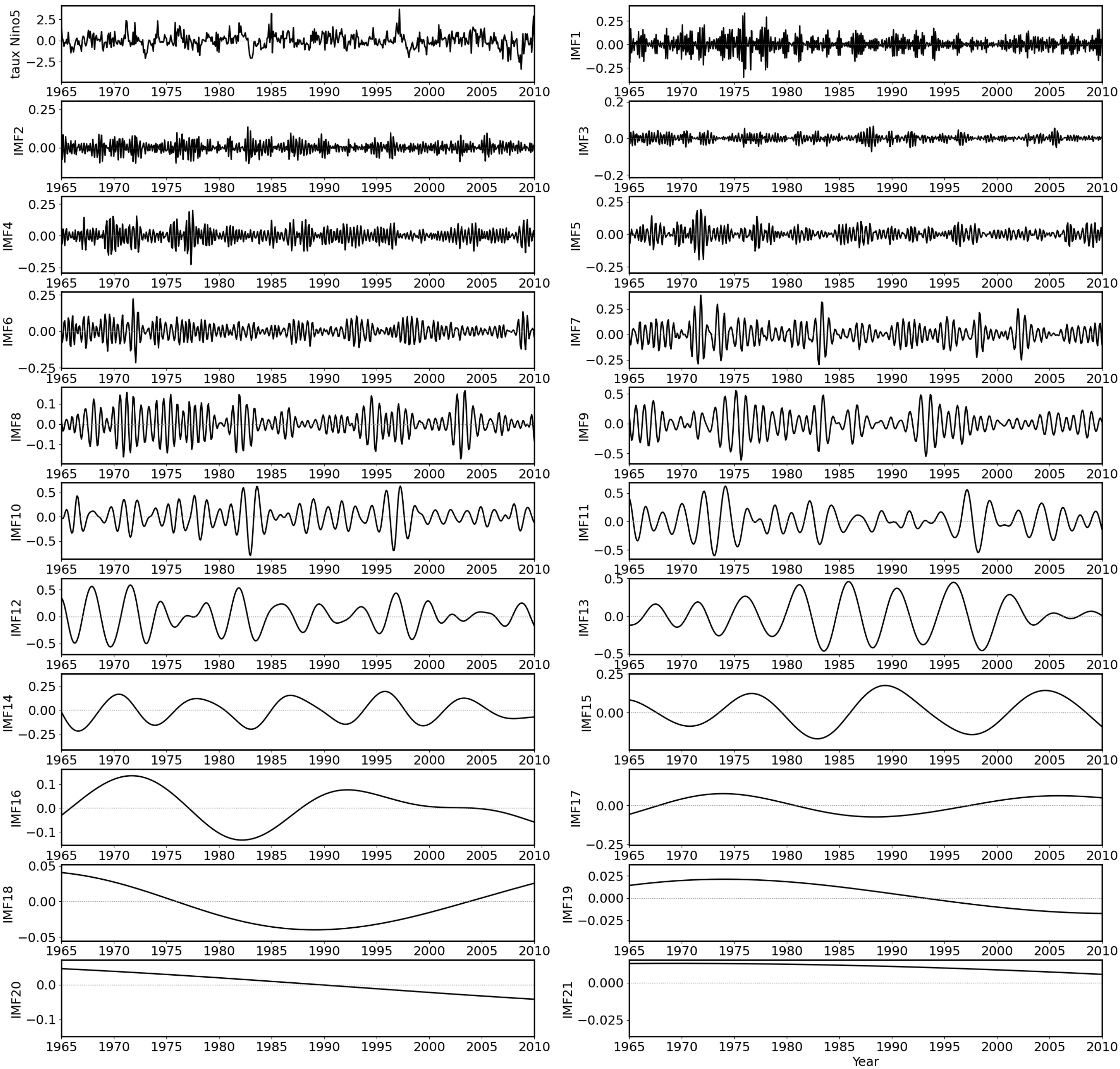

Figure S8. As Fig. S6 but for western Pacific τ_x (Niño5). For characteristic periods of IMFs see Table S1 (fourth column).

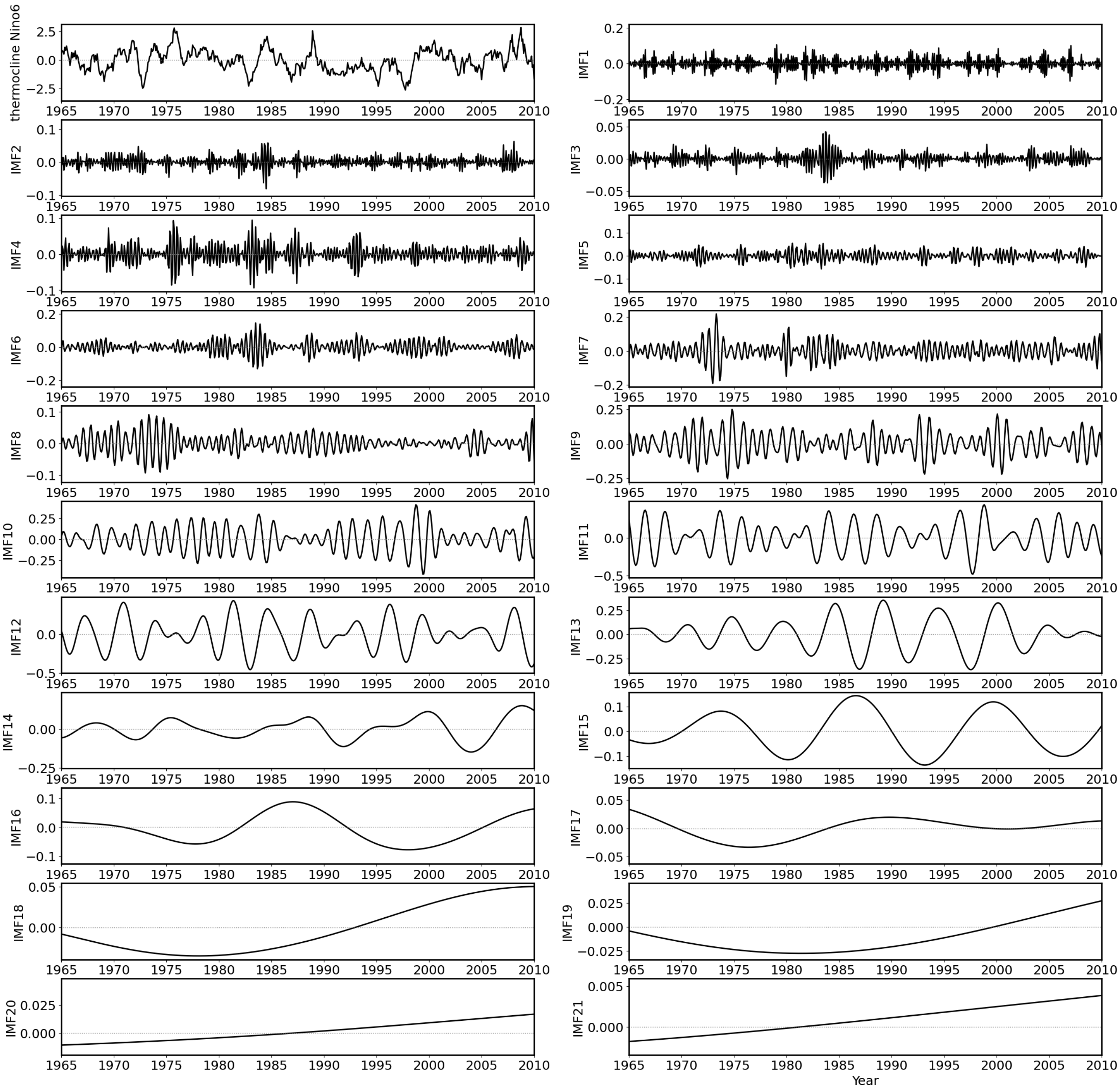

Figure S9. As Fig. S6 but for western Pacific off-equatorial thermocline depth (Niño6). For characteristic periods of IMFs see Table S1 (fifth column).

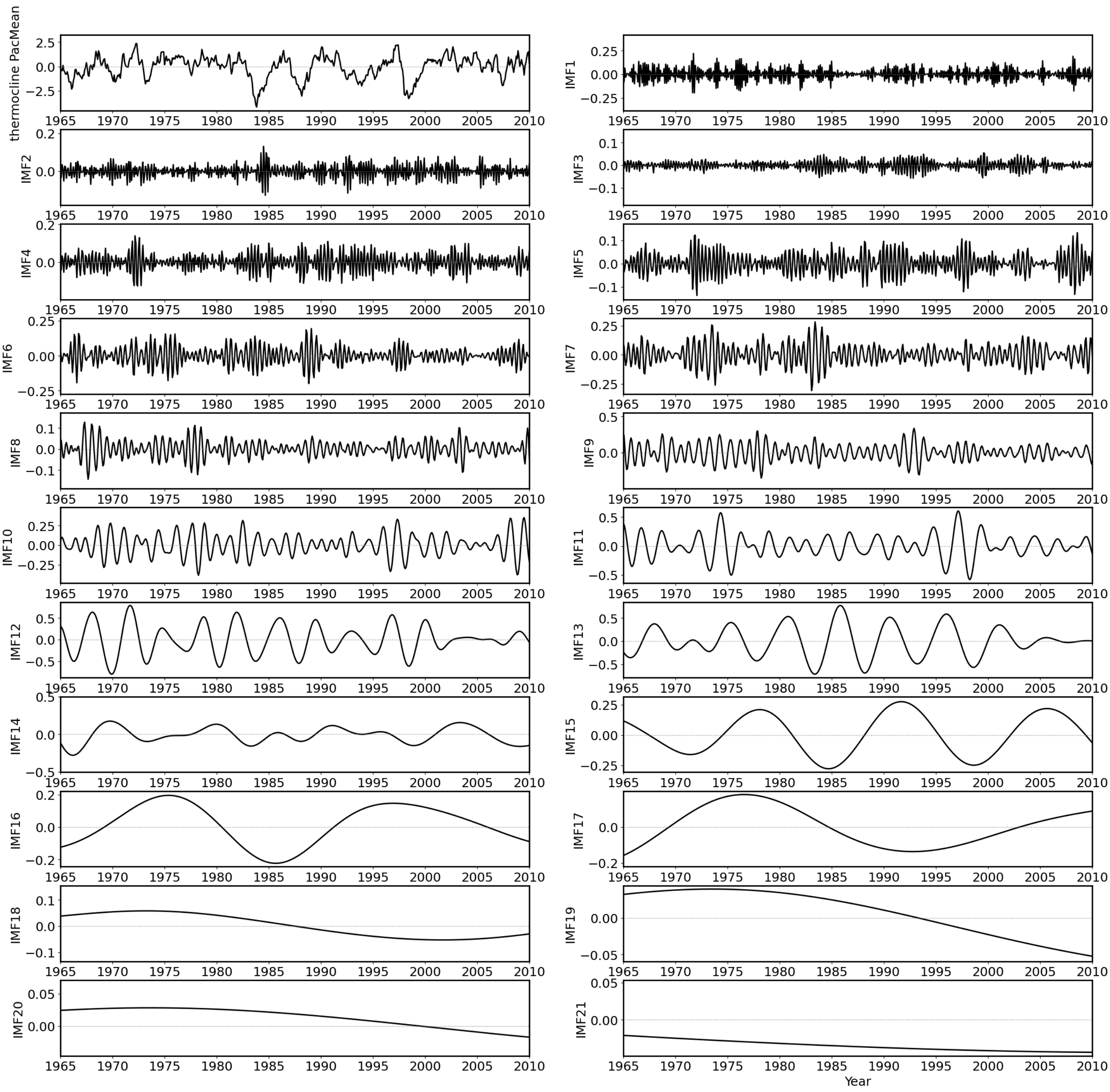

Figure S10. As Fig. S6 but for Pacific mean thermocline depth. For characteristic periods of IMFs see Table S1 (right column).

S.2 Supplementary Tables

	Niño3 SST	Niño4 τ_x	Niño5 τ_x	Niño6 thermocline depth	Pacific mean thermocline depth
IMF1	2.9	2.9	2.9	2.9	2.9
IMF2	3.1	3.2	3.0	3.2	3.1
IMF3	3.2	3.2	3.3	3.3	3.3
IMF4	3.8	3.7	3.8	3.7	3.8
IMF5	4.3	4.2	4.3	4.3	4.3
IMF6	5.2	5.2	5.2	5.2	5.1
IMF7	6.3	6.3	6.4	6.4	6.3
IMF8	7.5	7.5	7.5	7.5	7.6
IMF9	9.8	10	9.8	10	9.4
IMF10	15	15	14	14	14
IMF11	23	23	24	24	23
IMF12	39	39	37	37	40
IMF13	58	56	54	54	56
IMF14	89	85	90	90	97
IMF15	141	152	152	152	162
IMF16	206	274	239	239	225
IMF17	370	358	336	336	391
IMF18	590	624	582	582	622
IMF19	713	1120	879	879	1070
IMF20	1965	1988	1844	1845	1013
IMF21	2013	2013	1764	1764	2013

Table S1. Characteristic timescales of all IMFs for eastern Pacific SST (Niño3), central Pacific τ_x (Niño4), western Pacific τ_x (Niño5), western Pacific off-equatorial thermocline depth (Niño6), Pacific mean thermocline depth. All values are given as approximate average periods in months. For corresponding timeseries of each variable's IMFs see Figs. S6-S10. Note that IMF21 is a trend by definition and similarly IMF18-IMF20 have long timescale, thus periods of these IMFs are harder to establish using Hilbert transform (text below Eq. B4).

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15	PC16	PC17	PC18	PC19	PC20
IMF1	2.9	2.8	2.9	3.0	3.0	3.0	2.9	3.0	3.0	2.8	3.0	2.9	2.8	2.8	3.0	2.8	3.2	3.0	2.9	3.1
IMF2	3.2	3.2	3.1	3.1	3.1	3.2	3.1	3.2	3.1	3.1	3.1	3.2	3.1	3.1	3.2	3.1	3.2	3.1	3.2	3.2
IMF3	3.3	3.2	3.3	3.3	3.2	3.3	3.3	3.3	3.2	3.2	3.2	3.2	3.3	3.2	3.3	3.3	3.3	3.3	3.3	3.3
IMF4	3.7	3.7	3.8	3.8	3.8	3.7	3.7	3.8	3.7	3.8	3.7	3.7	3.7	3.7	3.7	3.8	3.7	3.7	3.8	3.7
IMF5	4.2	4.3	4.3	4.2	4.3	4.3	4.4	4.4	4.4	4.3	4.4	4.3	4.4	4.3	4.4	4.3	4.3	4.4	4.3	4.3
IMF6	5.2	5.2	5.2	5.1	5.2	5.2	5.1	5.1	5.2	5.1	5.1	5.2	5.1	5.1	5.1	5.1	5.1	5.2	5.1	5.1
IMF7	6.3	6.3	6.4	6.4	6.3	6.4	6.3	6.3	6.3	6.3	6.4	6.4	6.4	6.3	6.5	6.4	6.3	6.4	6.4	6.4
IMF8	7.6	7.5	7.5	7.5	7.5	7.6	7.5	7.6	7.7	7.5	7.6	7.7	7.4	7.6	7.4	7.5	7.4	7.5	7.5	7.6
IMF9	10	9.9	9.5	10	9.7	9.9	9.6	9.9	9.8	9.8	9.6	9.9	10	9.8	9.9	9.9	9.6	9.8	9.7	9.4
IMF10	15	15	15	15	14	14	15	14	15	15	15	15	15	14	15	15	14	15	15	15
IMF11	23	22	23	24	22	22	22	22	21	21	21	23	22	22	23	22	22	22	22	23
IMF12	38	36	38	34	37	37	38	35	37	36	37	33	38	35	34	36	34	35	34	35
IMF13	54	58	56	54	60	54	60	56	57	56	59	61	61	56	56	60	59	57	59	60
IMF14	84	99	97	96	86	87	97	91	97	96	94	90	90	98	81	93	86	100	92	83
IMF15	138	146	147	134	156	150	141	133	151	144	126	153	152	160	157	150	168	134	167	144
IMF16	267	276	247	221	210	245	245	243	229	310	284	240	206	218	228	244	220	262	240	271
IMF17	361	395	454	429	413	337	335	356	401	426	400	349	418	372	365	421	409	342	334	424
IMF18	636	725	617	576	591	555	616	525	573	518	623	595	540	499	547	594	680	631	640	467
IMF19	610	897	1056	754	907	890	826	890	903	873	793	862	951	986	1080	920	889	875	1011	1127
IMF20	1940	1928	2124	1869	1892	1864	1818	1791	1841	1704	1839	1802	2139	1151	1942	1901	1876	1881	2008	1503
IMF21	2012	2012	2013	1851	1757	1982	1943	2005	1853	1945	1888	1780	2013	2012	2013	2009	1988	1999	2012	2010

Table S2. Characteristic timescales of all IMFs for all 20 PCs. All values are given as approximate average periods in months. Note that IMF21 is a trend by definition and similarly IMF18-IMF20 have long timescale, thus periods of these IMFs are harder to establish using Hilbert transform (text below Eq. B4).

S.3 Simple Example for MEMD

As an example, we describe how MEMD works on a few simple periodic timeseries. We define four timeseries that have a shared angular frequency of $\pi/2$ with other harmonics or phase shifts added on top. We input these timeseries into the MEMD and expect the MEMD to isolate the shared mode with angular frequency of $\pi/2$ in all four timeseries, i.e., find the synchronised signal within the timeseries. We also expect MEMD to find other harmonics in the timeseries.

To do this, we construct the four timeseries as follows

$$\mathrm{x_{inp}} = \sin\left(\frac{\pi t}{2}\right) \tag{S.1}$$

$$\mathrm{y_{inp}} = \sin\left(\frac{\pi t}{2}\right) + \sin\left(\frac{2\pi t}{2}\right) + \sin\left(\frac{4\pi t}{2}\right) \tag{S.2}$$

$$\mathrm{z_{inp}} = \sin\left(\frac{\pi t}{2}\right) + \sin\left(\frac{3\pi t}{2}\right) \tag{S.3}$$

$$\mathrm{w_{inp}} = \sin\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) \tag{S.4}$$

where t is time and $\mathrm{x_{inp}}$, $\mathrm{y_{inp}}$, $\mathrm{z_{inp}}$, $\mathrm{w_{inp}}$ are timeseries with a common periodic signal $\sin(\pi t/2)$ and a few additional timescales or phase shifts. Thus, $\mathrm{w_{inp}}$ is the same as $\mathrm{x_{inp}}$ but 90-degrees phase shifted, whereas $\mathrm{y_{inp}}$ and $\mathrm{z_{inp}}$ have additional timesecales that are double, tripple or quadruple of $\mathrm{x_{inp}}$'s timescale. These four timeseries (Fig. S11, top left) are input into MEMD algorithm. The algorithm then returns 5 IMFs. IMF3 (Fig. S11, middle right) can be considered as the goal of this data, i.e., identification of common timescales across the 4 different timeseries/datasets, i.e., angular frequency $\pi/2$ (as mentioned above). The algorithm identifies the same mode in all four timeseries despite phase shifting or presence of other timescales in these simple timeseries. Such a mode is robustly identified across different parameter sweeps of MEMD (not shown). Thus, IMF3 can be considered here as equivalent of the ENSO's LF/QQ mode that has been shown in the past to exist across the tropical Pacific and a similar mode is identified again in the main text via MEMD as well.

IMF1 (Fig. S11, top right) represents the fastest 'waves' (shortest period/timescale) that we can find in $\mathrm{y_{inp}}$ and $\mathrm{z_{inp}}$, i.e., related to angular frequencies $3\pi/2$ and $4\pi/2$. The latter two frequencies are identified by the MEMD as similar thus they appear in the same IMF, although one could change the parameters of the MEMD algorithm to split the two modes into separate IMFs. However, that can then lead to splitting up other modes as well (especially IMF2), leading to unrealistic results (i.e., mode mixing; not shown). IMF2 (Fig. S11, middle left) shows intermediate angular frequency present in $\mathrm{y_{inp}}$, i.e., $2\pi/2$, but this IMF's output is not perfect, resulting in varying amplitudes of the wave throughout the analysis period, and thus IMF4 and IMF5 (bottom panels in Fig. S11) then compensate for the loss of amplitude in IMF2 in this case. Note that a longer timeseries somewhat helps mitigating this issue as any timeseries analysis tool has issues at the edges of the data and thus only data sufficiently far from the edges should be considered in analysis (there amplitude can be somewhat stable in IMF2). This means that longer datasets are preferred for MEMD analysis to ensure stability. Also, IMF4 and IMF5 should technically be zero (given the chosen input timeseries), but due to edge effects and other issues with (M)EMD method (see main text for details) they are still present though their amplitudes are small. This suggests that IMFs of the longest periods can sometimes be rather artificial constructs of the data and should be treated with caution especially when the trend of the data is essentially zero (as here or in the main text where trend has been removed prior to MEMD analysis).

This example only shows that signals that are well synchronized across timeseries will show up clearly in MEMD analysis, however other signals that exist in, e.g., only one mode (e.g., y_{inp}'s $2\pi/2$ wave) can be problematic as the method may struggle with keeping zeros in other timeseries (see IMF2). Then, leaking can occur both within, e.g., IMF2 and into other modes, causing mode-mixing again (like here IMF2 leaks into IMF4,5, especially at the edges). Similar issues can exist with trends as shown here. Thus, caution and verification with other methods is advised when using MEMD.

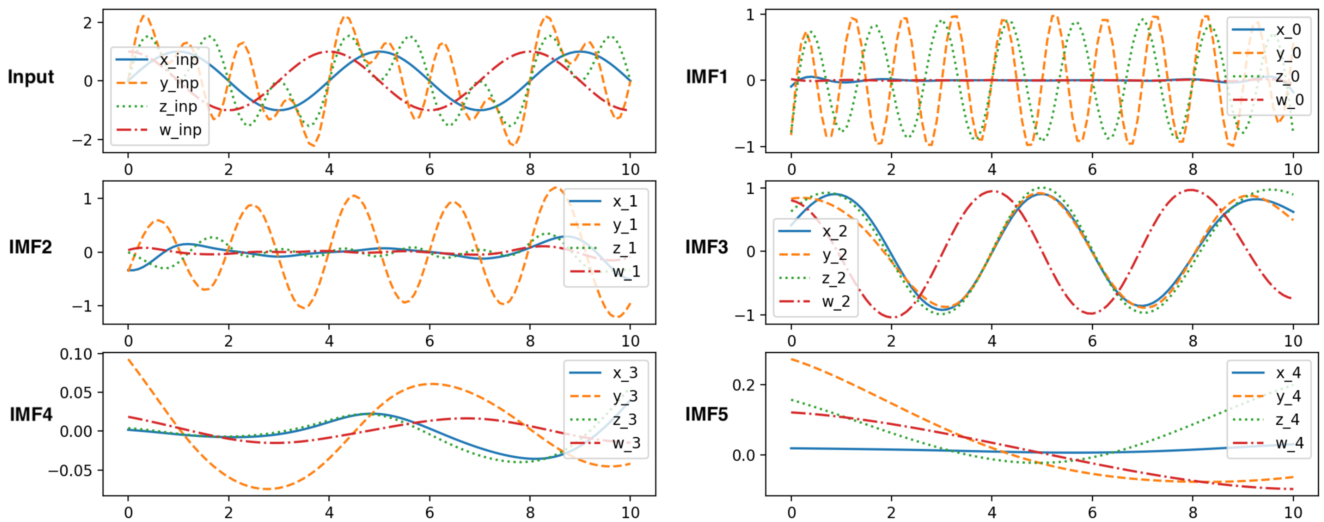

Figure S11. MEMD analysis of simple timeseries (Eqs. S.1-S.4). Top left panel shows input timeseries and the rest of the panels show the five IMFs that MEMD produces. IMF5 typically represents the trend of the data. See text for more details. Note that amplitudes of IMF4,5 are smaller than for IMF1,2,3 (i.e., y-axes are not the same across panels).