**Stratospheric intrusion depth and its effect on surface cyclogenetic forcing: An idealized PV inversion experiment**

by Barnes et al.

The manuscript improved during the last revision. However, I still do have suggestions to improve the manuscript. I think after this round of minor revisions the manuscript might be ready for publication.

**Penetration depth**

The authors argue in Experiment 2 in LL454 that a lower tropopause results in an increased static stability leading to increased rotation (decreasing negative vorticity) around the anomaly. However, the penetration depth \( H \) of an anomaly varies inversely with the ambient static stability \( \mathcal{N} \) around an anomaly (e.g. Martin 2013):

\[
H = f \frac{L}{\mathcal{N}} \quad \text{with} \quad L \text{ the horizontal scale of the anomaly.} \tag{1}
\]

I would ask the authors to insert potential temperature contours especially into Fig 9 and Fig 14 to confirm their argument and solve this contradiction. I would expect to see a lower static stability in cases with lower tropopause height since the cyclonic flow is stronger on the surface.

In Experiment 5 the penetration depth nicely varies with horizontal scale as expected following equation (1). I recommend to include the well-known concept of penetrations depth into the discussion of both, experiment 2 and 5.

The contradiction is well noted. The ambient potential temperature gradient is constant in the troposphere throughout all the experiments as defined by the static stability settings. However, it is the local static stability (local potential temperature) that is enhanced when the PV intrusion is closer to the surface, i.e., all values of the equation \( H = fL/\mathcal{N} \) are equal in all three cases of Experiment 2 since \( H, L, f, \) and \( \mathcal{N} \) are all held constant.

We have aimed to clarify this contradiction argument by speaking more directly to the various terms in the QGPV equation in the paper in our arguments of Experiment 2.

In order to keep the figures clean, we have not shown the potential temperature contours as suggested, however we have quoted values of increased potential temperature gradients within the intrusion.

\[
\zeta + f = q - f \frac{\partial}{\partial z} \left( \frac{\partial \mathcal{E}^{-1}}{\partial z} \theta \right) \tag{1}
\]

The environmental potential temperature gradient is set as constant since the tropospheric and stratospheric static stability values are kept constant throughout Experiment 2 (as explained in Section 2.2). Therefore, the term \( \frac{\partial \mathcal{E}}{\partial z} \) in Equation 6 does not change between the high and low tropopause cases. As a result, the change in relative vorticity is controlled by a local change in the potential temperature gradient (\( \frac{\partial \mathcal{E}}{\partial z} \)). Calculation of this term within the intrusion shows that there is an increase in the local potential temperature gradient between three scenarios in Experiment 2. In fact, in the center of the intrusion at the height of the maximum meridional velocity, the potential temperature gradient (\( \frac{\partial \mathcal{E}}{\partial z} \)) increases from 0.44K.m\(^{-1}\) in the higher tropopause scenario to 0.47K.m\(^{-1}\) in lower tropopause scenario. Since \( \frac{\partial}{\partial z} \left( \frac{\partial \mathcal{E}^{-1}}{\partial z} \right) < 0 \) and both \( q \) and \( f \) are negative in the Southern Hemisphere, it follows that an increase in \( \frac{\partial \mathcal{E}}{\partial z} \) will result in a decrease in relative vorticity in the Southern Hemisphere. The more tightly packed potential temperature contours within the intrusion (higher static stability) in the scenario where the dynamical tropopause is closer to the surface results in a decreased cross-sectional relative vorticity compared to the higher tropopause scenario.
Specific and technical corrections

L13: high PV (large negative PV) -> remove information about negative PV anomalies in abstract or mention focus on southern hemisphere. otherwise confusing
Removed negative reference in the abstract as per suggestion

L22: add that after fact: is the fact that PV
Change applied as per suggestion

L60: Bierly confirmed/showed
Change applied as per suggestion

L128-L130: piecewise PV inversion is no methodology of solving equation (1), but more an extended approach thanks to the additive behaviour of PV anomalies. Furthermore the different techniques studied by Davis 1992 are only necessary for PV inversion under nonlinear balance (since the equations are nonlinear) and not quasi-geostrophic balance. Under quasi-geostrophic balance the flow fields from different piecewise inversion are additive and do not depend on the different approaches suggested by Davis 1992. I hence suggest to remove this paragraph.
Change applied as per suggestion

L140: remove „using a piecewise numerical approach“
Changes applied as per suggestion

L157: define AGL the first time using it
AGL first used and defined on L119

Eq5: x-xi -> x-x_pos
Changes applied as per suggestion

L419: systemic -> systematic?
Changes applied as per suggestion

L519: lesser -> less
Changes applied as per suggestion
L551: ie. By -> i.e. by Changes applied as per suggestion
Changes applied as per suggestion

L573: ie. That -> i.e. that
Changes applied as per suggestion

L607: result -> results
Changes applied as per suggestion

L609: why although? there is no contradiction, is it? (L445)

LL610: The authors state that „The larger magnitude relative vorticities induced by thinner intrusions are the result of the circulation with lower velocity being confined to a smaller horizontal region around the anomaly.“ However, from Fig14 I identify larger magnitude in relative vorticity for broad intrusions.. the rel. vorticity is more negative for broad intrusions and hence the cyclonic circulation is stronger. Please clarify! Especially in L638 the authors write „Enhanced surface cyclonic rotation is also induced by the broader PV anomaly with increases in the surface relative vorticity. “ and contradict themselves.

The first quotation referred to by the reviewer ("The larger magnitude relative vorticities induced by thinner intrusions are the result of the circulation with lower velocity being confined to a smaller horizontal region around the anomaly.") is in reference to the mid-tropospheric relative vorticity, whilst the second ("Enhanced surface cyclonic rotation is also induced by the broader PV anomaly with increases in the surface relative vorticity.") refers to the differences in relative vorticity at the surface. Relative vorticities are enhanced for thinner intrusions in the mid-troposphere whilst are limited at the surface, whilst the reverse is true for broader intrusions. We have clarified this by adapting the wording of this section to explicitly say which vertical levels we are referring to.
L670: i.e. -> i.e.
Changes applied as per suggestion

L704: I would not state key finding, since all results are not knew but nice to have summarized in one study.
Changed to “The relative contribution of different factors to surface cyclogenetic forcing is highlighted in this study.”

L715: remove of before with
Changes applied as per suggestion

References

Martin, Jonathan E. *Mid-latitude atmospheric dynamics: a first course*. John Wiley & Sons