Reply to the Reviewers' comments

General

First, we would like to thank both reviewers for their thoughtful comments. At the heart of both reviews is the difficulty of separating PV conserving from PV non-conserving stability and vorticity changes in the diagram. It is argued that this information is not immediately apparent from the diagram. Further, both reviewers point out that PV conserving and non-conserving stability and vorticity changes can occur simultaneously, which is also not distinguishable in the diagram.

We argue that all the information requested by the reviewers is implicitly contained in the diagram and the separation into PV conserving and non-conserving stability and vorticity changes becomes apparent with only some minor modifications presented on the next pages.

Further, we argue that it is even possible to quantify the relative contributions by non-conserving stability and non-conserving vorticity changes to PV non-conservation without the need for diabatic model output. This is a key advantage of the diagram. Our motivation to study the PV composition and adiabatic and diabatic changes will now be presented at the end of this reply document. We hypothesize that the PV composition, i.e., its partioning into stability and vorticity, has important consequences for the downstream development. The diagram can help understand the adiabatic and diabatic contributions to the formation of a specific PV composition and its consequences for the downstream development.

On the next pages we outline how we suggest to adapt the diagram to make the requested information more accessible. Much of these explanations were missing in our original manuscript but the required changes are straightforward. Afterward, we also provide a point-by-point reply.

Kind regards
The authors

Suggested revisions

A vorticity-and-stability diagram to study the temporal evolution of the PV composition:

We are interested in the nature of a change in the PV composition (in terms of its stability and vorticity). In the illustrative example in Fig.1, PV is changing between two times from 0.5 PVU to 1 PVU (black vector). The accompanying changes in stability and vorticity can be decomposed into PV conserving changes of stability and vorticity (change along the red hyperbola) and PV non-conserving changes of stability and vorticity (change along the gray hyperbola). In this example, obviously both contribute to the change in the PV composition.

Without loss of generality, we can define a rotated coordinate system (green coordinate system in Fig. 1) that is locally tangential to conserving stability and vorticity changes and with its second axes orthogonal to it, that is, the second axis is tangential to PV non-conserving stability and vorticity changes (the gray hyperboles). This procedure is standard and similar to what is done in differential calculus to estimate the derivative of a non-linear function using the slope of a line tangent to a point.

To quantify contributions by conservative and non-conserving stability and vorticity changes, which is requested by both reviews, the scalar products between the vector pointing into the direction of the change in the PV composition (black vector) and each of two base vectors of the local coordinate system (green vectors) is computed.

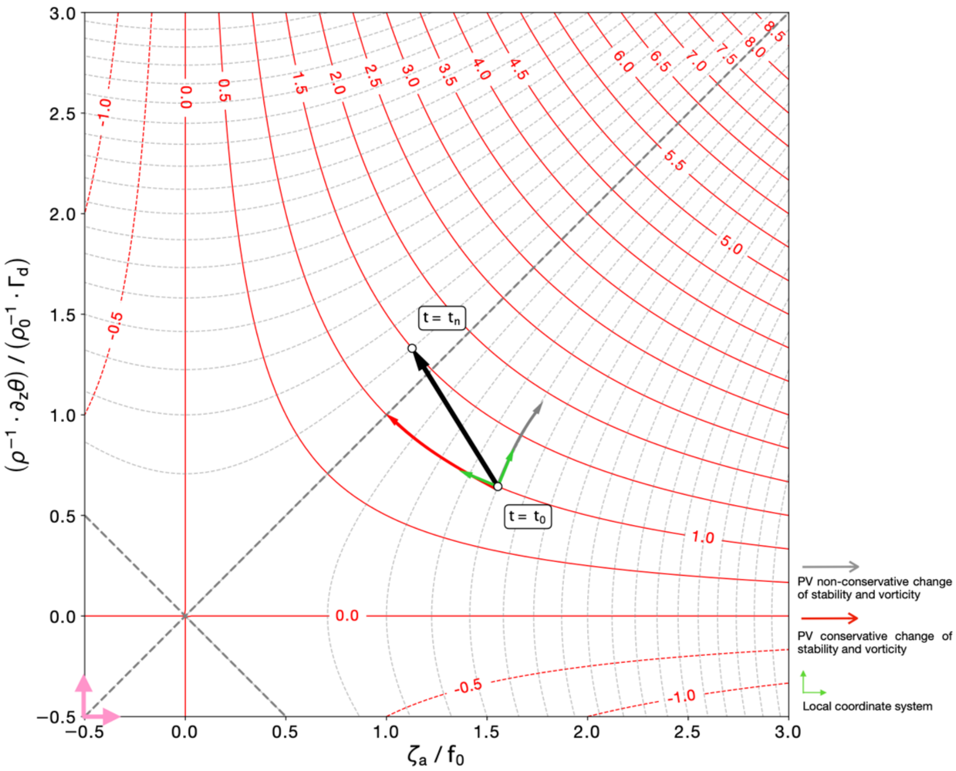

Figure 1: A change in the PV composition between two time steps (t_0 and t_n) can be decomposed into a change from PV conservative stability and vorticity changes (red) and PV non-conservative stability and vorticity changes (gray). To this end, a local coordinate system is defined (green) that is tangential to the conservative and non-conservative change (green). The

scalar products between the change vector (black) and the base vectors of the local coordinate system (green) quantify the relative contributions by PV conserving and non-conserving stability and vorticity changes. The scalar products between the vector tangent to the non-conservative change (gray) and the two base vectors of the stability-and-vorticity diagram (pink vectors lower left corner) quantify the relative importance of the PV non-conserving stability change compared with the PV non-conserving vorticity change. Axes are unitless. The red and gray vectors are curved only for illustration.

The nature of the decomposed change is now visualized in the local coordinate system (green in Fig. 1). If the vector in the local coordinate system in Fig. 2 points

- upward, it indicates a stability increase and vorticity decrease, which conserves PV;
- downward, it indicates a stability decrease and vorticity increase, which conserves PV;
- to the right, it indicates a PV increase;
- to the left, it indicates a PV decrease.

For the PV non-conservative change (vector points to the right or to the left), the relative importance of non-conservative stability compared to non-conservative vorticity change is indicated by the color of the dot (Fig. 2). To obtain it, we compute the scalar products between a vector tangent to the direction of PV non-conservation (gray hyperbolas in Fig. 1) with the two base vectors of the coordinate system of the original diagram (pink vectors in lower left corner in Fig. 1). Subsequently we compute the fraction of the two values. A low value (blue color) indicates the dominance of non-conserving stability change over non-conserving vorticity change; a high value (red color) indicates the dominance of non-conserving vorticity change.

As pointed out by the reviewers, simultaneous changes are common. If the vector in local coordinates, as in this example, points to the upper right in an approximate angle of 45° it indicates that adiabatic and diabatic stability and vorticity changes occur simultaneously and both change the PV composition equally. The length of the vector indicates the magnitude of the change. We also see from the color coding that for the non-conservative change (i.e., the PV increase) the non-conservative stability change (hence a diabatic stability increase) dominates over non-conservative vorticity change (blue dot in Fig. 2).

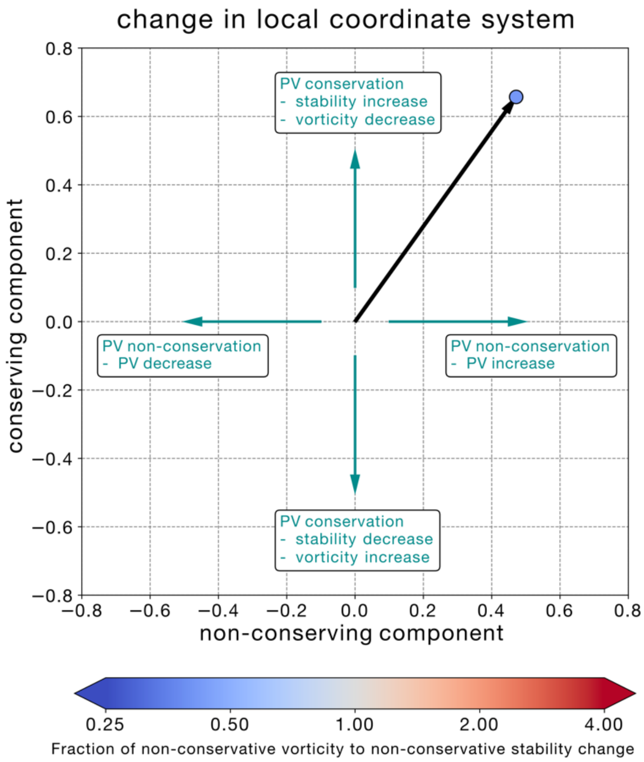

Figure 2: Change in the PV composition decomposed into contributions by PV conserving and PV non-conserving stability and vorticity change in a coordinate system locally tangent to conservative and non-conservative changes in Fig. 1. The

color coding indicates the relative contribution to the non-conservative part of the change by non-conservative vorticity relative to non-conservative stability change. The estimation is possible without any diabatic model output. The interpretation of the orientation of the vector is shown in the diagram. See text for details.

The above procedure is repeated for several time steps. The next figure (Fig. 3) shows a hypothetical time series with ten steps and ten changes in the PV composition. The two panels on the right show the nature of changes in the local coordinate system in a compass-like plot (Fig.3B) and also as a time series, which is simply obtained by adding each value to the preceding value (Fig. 3C).

From Figure 3 the nature of the change in the PV composition (as requested by the reviewers) becomes clear at every time step. For the first three timesteps (labelled with “1”), the vectors point purely to the left (Fig. 3B and C), which indicates a PV decrease, and the blue coloring suggests the dominance of non-conservative stability change over non-conservative vorticity change (hence we tend to call it a diabatic stability-dominated decrease of PV). For the next three timesteps (labelled with “2”) the vectors point upward and slightly to the right indicating an almost conservative change in the PV composition (conservative stability increase and vorticity decrease) with a minor non-conservative PV increase (vector points slightly to the right) that is almost equally due to non-conservative stability and vorticity changes (gray coloring of the dots). This is followed by two timesteps (labelled with “3”) where the change in the PV composition is still due to both conservative and non-conservative stability and vorticity changes but because the vectors point stronger upward than to the right, our interpretation is that the adiabatic change dominates over the diabatic change of the PV composition. The non-conservative change is dominated by a diabatic vorticity increase (red dots). Finally, for the last two timesteps (labelled with “4”), the vectors point to the lower right, which indicates a diabatic PV decrease but again a combination of diabatic and adiabatic changes of the PV composition occurs. The orientation is about 45 degrees, which suggests equally strong contributions by adiabatic and diabatic changes to the PV composition. The diabatic PV increase in this period is dominated by diabatic vorticity increase (red coloring of the dots), while the adiabatic change of the composition is a stability decrease combined with a vorticity increase (downward component of the vectors).

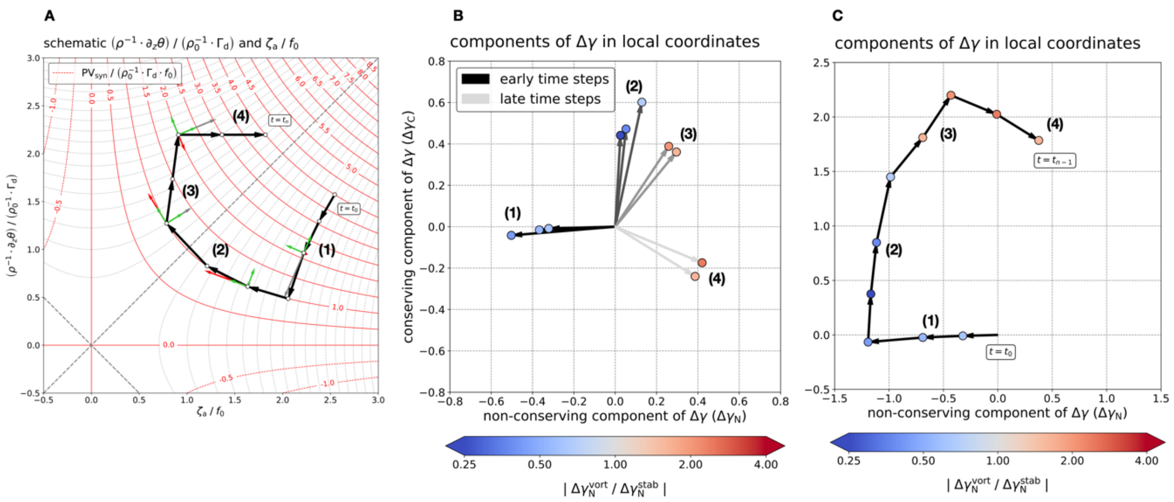

Figure 3: Ten timesteps of a hypothetical change in the PV composition as seen in the stability-and-vorticity diagram (A), the nature of the changes in the local coordinate system (B), and the temporal evolution of the change in the local diagram (C). In the middle and right panel, if the vector points upward, it indicates a stability increase and vorticity decrease, which conserves PV; downward, it indicates a stability decrease and vorticity increase, which conserves PV; to the right, it indicates a PV increase; to the left, it indicates a PV decrease. The coloring indicates the dominance of nonconservative vorticity (red) or non-conservative stability (blue). Fig. 2 helps to understand the importance of the vector orientation in the middle and right panels. Δγ indicates the change of the PV composition.

In our opinion, the diagram thus provides unique insight into the PV composition, the change of the composition and the fractional contributions by adiabatic and diabatic stability and vorticity changes. There is no need for generating additional model output (diabatic heating or momentum tendencies are not required). The revised diagram thus provides the information requested by the reviewers.

In the revision, we also suggest applying the revised diagram to the idealized WCB and the real-case WCB simulations and suggest removing the final case study.

The only remaining caveat is the use of the conventional large-scale assumption that PV can be reasonably approximated by considering only the vertical component of vorticity, which we called PV_{vert} or vertical PV, admittedly an unfortunate and misleading choice. Further, the diagram alone does not give information on the process level (diabatic tendencies of microphysics, radiation, turbulence etc), which we added as a separate panel.

The idealized WCB

For the idealized WCB, the nature of the change of the PV composition is shown below. This diagram will be discussed in the revised manuscript alongside that of the real-case WCB in detail.

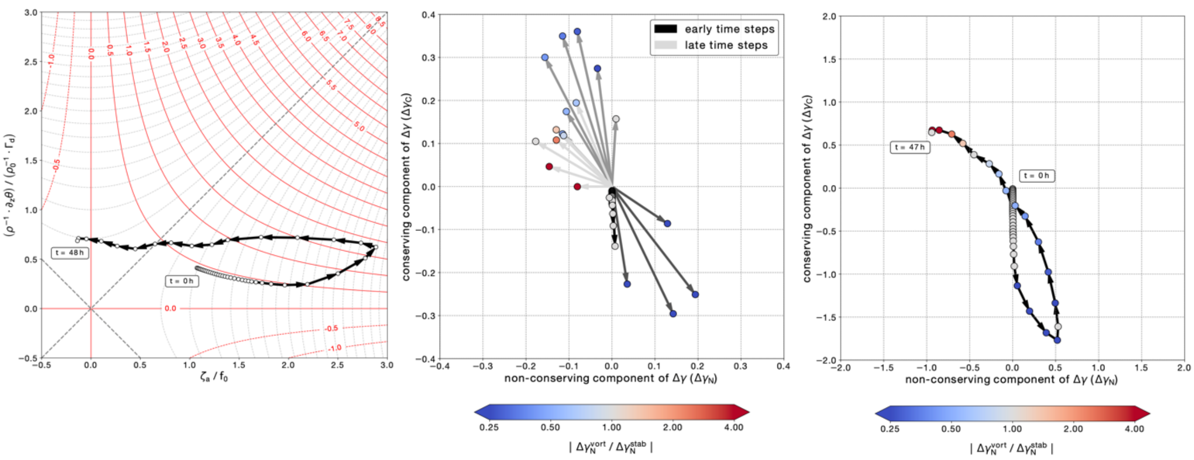

Figure 4: Idealized WCB and accompanying change in the PV composition as seen in the stability-vorticity diagram and the local coordinate system. See text for details.

Motivation behind the diagram

Concerning the lessons that can be learned from such a diagram, we argue that the diagram is essentially a means to study the PV composition as an alternative to PV inversion. The now modified diagram allows to quantify how a given PV anomaly is composed by stability and vorticity and how the temporal evolution of adiabatic and diabatic stability and vorticity changes leads to a specific PV composition. We hypothesize that the PV composition found in warm conveyor belt (WCB) outflows exhibits a large case-to-case variability, which could explain why some WCBs are pre-cursors to downstream ridge building and block formation, while others are pre-cursors to downstream cyclone development. Arguably, the PV composition will have an influence on the downstream development. Most recent studies focus however exclusively on diabatic modification of the PV composition, the diagram adds the adiabatic change to it.

Point-by-point replies:

Review #1

General comments

Review: One certainly cannot dispute the usefulness of studies that look in detail at the processes that change PV, e.g., in forecast models, as are exploited in the later part of this paper. But my reservation about this paper is over whether or not it provides a genuine advantage for such studies. The particular shortcoming of the approach proposed here is that the diagram does not by itself provide any information on the physics operating.

Using a highly abbreviated notation, let the PV be Q, the absolute vorticity be Z and the static stability be S. Using dQ to denote Lagrangian change in Q, for example, we have

dQ = S dZ + Z dS

Now suppose that dZ = dZ_P + dZ_D where the first term on the right-hand side is due to non-conservative physics and the second is due to conservative dynamics, with corresponding notating for dS. Then S dZ_D + Z dS_D = 0 because there is no change in Q is conserved under the effects of conservative dynamics alone.

The diagram gives us information about dZ and dS (i.e. two pieces of information). Even though we have the constraint S dZ_D + Z dS_D = 0 we cannot determine dZ_P and dQ_P separately -- so whether or not there are non-conservative diabatic processes (acting on temperatures) or non-conservative mechanical processes (acting on velocities) cannot be determined.

To me then, it seems that the behaviour seen in the diagram can, of course, be explained in terms of the physical processes acting if those are known, but the information given in the diagram is NOT sufficient to determine what those processes are (i.e. how they are partitioned between diabatic or mechanical in sense used above). In your case studies described in Sections 3.2 and 3.3 you have (and present) the detailed information about how different physical processes contribute to PV changes -- how does the use of the diagram add anything?

Reply: Thank you for this detailed comment, which is highly appreciated.

In the revision, we show, as requested by the reviewer, how we can use the diagram to determine all four contributions dZ_D, dZ_P and dS_D and dS_P separately by defining a local coordinate system.

We agree that dS_P and dZ_P alone give no information on the level of individual non-conserving processes – say turbulence, microphysics, evaporation – but their aggregated influence as measured by dS_P and dZ_P can be determined from the diagram. There is no need for additional model output, which can be an advantage because many climate models do not provide diabatic tendencies on the level of individual processes or these are complex to obtain.

As stated above, the separation into conservative and non-conservative stability and vorticity changes is possible in the diagram and allows to understand the formation of a specific PV composition, which can either be more thermally (stability) or circulation (vorticity) dominated, and we hypothesize that this has important consequence for downstream development following strong WCB activity.

Review: Abstract: very long -- seems to miss the point of an abstract which is to provide a brief summary of the aim, methodology and findings of the paper. In fact the text in general is overlong -- there is a lot of background material.

Reply: We will condense the abstract in the revised version and put a focus on the idea underlying the diagram.

Review: l10: 'hyperbolic' -- term will be meaningless to reader without explanation.

Reply: Will be moved into the method section.

Review: l61: '"latent vorticity" generation' -- first time I had come across this term -- which essentially seems to mean forcing of PV by diabatic processes (with the 'direct' effect, so to speak, on the temperature/stability) together with the very familiar principle that the partitioning of PV between relative/absolute vorticity and stability can change through purely reversible conservative processes. The term seems to be very rarely used and, with all respect to Chagnon and Gray (2009), I'm not convinced that it helps general understanding to perpetuate it.

Reply: Will be removed.

Review: l62: 'adjusts to a new balanced state in the process of hydrostatic-geostrophic adjustment ... during which inertia, gravity and sound waves radiate away from the heating perturbation' -- there is a question -- perhaps it is a matter of taste -- about whether it is appropriate to describe evolution of a balanced flow as a continuous process of geostrophic adjustment. One subtlety is that the amount of emitted wave activity is determined not just by the difference between the two states A and B, say, at different times, but by the time that elapses between A and B (see Vanneste 2013). There are some advantages to restricting the term geostrophic adjustment to an initial value problem or a problem with 'impulsive' forcing.

(But as I have noted -- this is partly a matter of taste -- I'm not insisting on a change.)

Reply: We do not have a clear preference and are happy to change it.

Review: l86: 'moist diabatic processes' -- not all processes that affect PV, even in the troposphere, are moist.

Reply: We fully agree. When we referee to moist diabatic processes we use it intentionally to limit the discussion to processes that include phase changes of water. Otherwise, diabatic processes contains turbulence, radiation, etc.

Review: Figure 1: My understanding is that the colours of the dots here are not providing 'extra' information -- they are simply displaying information that could be deduced from the diagram -- since (using the notation I have introduced above) what you are indicating is | S dZ / Z dS | -- which can be deduced from the position in the diagram and the slope of the curve.

(This is not a criticism of the use of the colours -- but I think it is important to be clear on what is 'new' information and what is not.

Reply: Yes, we introduced it only to help guiding the eye. In the revised version, the new panel on the right side uses colored dots, which introduce a new information. It is the relative contributions by PV non-conserving stability and non-conserving vorticity changes.

Review: l99: 'vertical component' -- I realise that taking account only of the vertical component of absolute vorticity is a useful simplification, and I don't have any particular problem with that, but I do think that the term 'vertical component of PV', which you use subsequently at various points in the paper, is a unfortunate. PV is a scalar, so it doesn't have a vertical component in the sense that absolute vorticity, as a vector, has a vertical component. Your terminology muddles use of 'component' with respect to a vector, with the more general use of 'component' as meaning 'part of'. It is not a serious problem, but it is not very elegant or precise.

Reply: We fully agree, this was a very unfortunate choice. We will define it properly (as an approximation that considers only the vertical component of vorticity) in the revised manuscript and name it accordingly.

Review: Figure 3 caption: 'vocticity'.

Reply: Will be corrected.

Reply: l229-31: goes back to my earlier comment re 'geostrophic adjustment' -- evolution of the balanced state does not require substantial emission of gravity waves -- it may or may not. A better statement in my view would be something like 'the vorticity decrease occurs as part of the evolution of the balanced state (under conservative dynamics)'. I'm not convinced that the 'geostrophic adjustment' sentence is needed. ', which seems unlikely' could simply be added to the previous sentence. Certainly 'desired' is not the correct word to use.

Reply: The geostrophic adjustment sentence is not needed and will be removed.

Reply: l339: 'In the absence of diabatic processes, this vorticity reduction and stability increase (i.e, column shrinking) would occur in tandem to conserve PV. However, the lesson learned from the vorticity-and-stability diagram is that it seems as if the large-scale divergence drives a vorticity reduction, but the diabatic and adiabatic influences on static stability are engaged in a tug-of-war, such that PV is not conserved.' The first sentence, of course, is simply describing PV conserving dynamics, in the absence of, say, diabatic processes. If diabatic processes act then there will, unless the diabatic forcing term in the PV equation is zero, be a change in the PV. Some of this change will appear in vorticity, some will appear in static stability -- that is all well known and it depends on non-local effects -- it can't be determined simply from what happens in a single air parcel. I don't really see how the diagram is helping -- apart from showing that the two quantities change -- where does one go from that?

Reply: The revised diagram allows to quantify the relative contributions of conserving and non-conserving stability and vorticity separately. Illuminating the formation pathway (incl. conserving and non-conserving change) to a specific PV composition can help to understand the subsequent evolution of the upper-level flow field. See our main reply at the beginning of this document. We also recommend computing a large ensemble of air parcel trajectories, not a single one, and to either average (if the spread is reasonably small) or depict the evolution in terms of a density in the diagram.

Point-by-point replies:

Review #2

Review: This paper takes a potential vorticity (PV) perspective on atmospheric dynamics. Given that PV is broadly speaking a product of static stability and absolute vorticity, a material rate of change of PV can occur through non-conservative terms in either the heat equation or the momentum equation or both. However, it is well possible that both static stability and vorticity suffer a non-zero material rate of change, but PV is, at the same time, conserved: this is exactly what happens in the event of purely conservative flow with vortex stretching.

Reply: Yes, we fully agree. The first six hours of the idealized WCB are such a nice example. During this period, stability decreases, and vorticity increases, and PV is conserved. This is of course not new, but the exact PV composition and the change of the PV composition along a WCB was unknown so far.

As we argue above, the diagram does allow to disentangle conservative from non-conservative stability and vorticity changes and - with some minor modification - it even allows to quantify the relative contributions by non-conservative stability and vorticity changes without the need for extra model output. The diagram gives a complete picture of how the PV composition changes and illuminates the nature of the change (diabatically or adiabatically or a combination) by using a locally tangent coordinate system at every time step.

We hypothesis, that the PV composition in the WCB outflow has some important consequences for downstream development.

Review: The authors analyse the situation with the help of a novel diagram that represents the motion of an air parcel in a two-dimensional phase space spanned by absolute vorticity and static stability. This is an interesting approach. At the same time, I think that the paper does not live up to the expectations, and it seems to me that essential aspects of "adjustment to balance" need to be discussed more lucidly in order to make this a useful contribution to the literature. In particular, it seems to me that there is a fundamental flaw in the argument. Consider the following thought experiment which was originally suggested by (I believe) M. McIntyre and/or B. Hoskins quite some time ago (sorry, I cannot find the respective reference). Assume that initially a parcel is instantaneously being subject to differential heating such that its static stability increases; the point in phase space would move straight upward. The ensuing adjustment process is thought to be adiabatic such that the parcel moves along one of the red hyperbolas. During this adjustment process, part of the original material increase in static stability is reduced and converted into a material increase of absolute vorticity (such that PV is conserved during the adjustment process). Where exactly the point ends on the diagram during the adjustment process essentially depends on the ratio of static to inertial stability and the aspect ratio of the heating (see, e.g., the work of Eliassen 1952). Of course, in reality the (initial) diabatic change and the (ensuing) adjustment process cannot be separated from each other, rather they occur more or less simultaneously. In addition, the occurrence or absence of inertio-gravity waves depends on the time scale during which the initial non-conservative process is applied.

Now consider a second thought experiment where initially there is only an (impulsive) non-conservative material tendency on absolute vorticity, followed by the adiabatic adjustment process. Both thought experiments may lead to the same end point in the phase space diagram. Thus, considering only the change of the point in phase space from the initial to the end state does not really tell us anything about the nature of the non-conservative processes – they may be diabatic (non-conservative heat equation), friction (non-conservative momentum equation), or a mixture of both.

Reply: We really appreciate the reviewer's input and thank you for it. In your thought experiment, it is assumed that the heating pulse and the ensuing adjustment occurs below the model (output) time step, otherwise the development described in your thought experiment should be captured by the trajectory in the diagram.

We are convinced that it is possible to quantify the nature of the change in the PV composition, but we apparently missed to make it more apparent. Several unfortunate choices we made to describe our reasoning (such as the use of the "driven" or unclear separation between conservative and non-conservative changes) did not work in our favor. But it is possible to quantify the relative contributions of conservative and non-conservative processes because we can define a local coordinate system that is parallel to PV conservative changes in stability and vorticity and orthogonal to PV non-conservative stability and vorticity changes. The projection into this local coordinate system exactly allows to explore the two relative contributions. Further, the scalar product between the vector pointing along non-conservative axes and the base vector of the vorticity-and-stability diagram allows to quantify the relative importance of non-conservative stability and non-conservative vorticity changes. All of this is possible without the need for additional diabatic model output.

Review: For this reason, I cannot follow the basic argument that underlies the reasoning of this paper. The argument first occurs on line 110: "…. PV changes in regions where grey hyperbolas are oriented more vertically …. tend to be driven by changes in static stability." Not accounting for the problem that I am not sure whether "changes in static stability" here are meant to be conservative or non-conservative, I think that any such statement cannot be made based on the trajectory of the parcel on the diagram alone.

Reply: With respect to the gray hyperbolas, we referred to non-conservative stability change, but we agree that this was not well described. With the now introduced local coordinate system our arguments should become much clearer.

Review: In the end it does not become clear to me what we have really learned from the analysis using this novel phase diagram. As far as I understand the text, the authors themselves are not very clear about that, and this materializes in the fact that the abstract is very long and very detailed. If one has so many results to report, this raises the suspicion in me that there is not really any true result.

Reply: As we outlined above, the goal is twofold. We want to explore the temporal evolution of the PV composition in terms of stability and vorticity because we assume that it has some important consequence for downstream development and that there is a high case-to-case variability. Second, the diagram allows to disentangle contributions by conservative and non-conservative stability and vorticity changes to the change in the PV composition. This is in our opinion a powerful and unique diagnostic. The motivation will be clarified in the revised introduction.

Review: I had a problem with this manuscript in that I could sometimes not really evaluate the validity of individual statements because I did not fully understand them. For instance, one should very carefully distinguish between (1) observed (material) tendencies in vorticity and static stability (which may be due to either conservative or non-conservative processes) and (2) non-conservative (material) tendencies in vorticity and static stability (which could be obtained by analysing the corresponding non-conservative terms in the momentum and heat equation (although the authors consider this to be beyond the scope of the paper).

Reply: Yes, the revised diagram exactly provides this information. It allows to distinguish conservative from non-conservative stability and vorticity changes. We did not properly separate the two in our original manuscript.

Review: Another important concern of mine are the many occurrences of formulations (A "drives" B) that suggest a direction of causality where (as far as I can tell) the authors do not provide any prove of such causality. I suggest to simply replace the word "drive" or "driven" by a more appropriate word, and often this more appropriate formulation would simply be "A is associated with B". To give an example: I could not follow your interpretation of the diagram in Fig 3d: what do you mean when you say that a PV-ver change is "driven" by …, and what does this mean? You should be more explicit here. Especially it is not clear to me whether a "change" in stability or vorticity is meant to be conservative or non-conservative.

Reply: We will take this into consideration if we are invited to a revision. Often, we are referring to the dominance of one process over the other and use the word "driver" for the leading diabatic process instead of "dominating process". We agree that we cannot guaranty strict causality and we will use more appropriate wording following your recommendation. In the revised diagram we will strictly refer to conservative or non-conservative change throughout the manuscript.

Review: PV-ver is a strange variable. What do we know about it? It is not necessarily materially conserved for conservative flow. This is dangerous, since the impact of non-conservative processes is at the heart of your analysis. A few lines later (and in the remainder of the text) PV-ver and PV are essentially treated as synonymous….

Reply: Considering only the vertical component of vorticity is a widely used approximation in large-scale dynamics. We recommend ensuring that this approximation holds true whenever the diagram is used. In the future, a higher-dimensional space could be used, but for not we restrict the diagram to two dimensions.

Review: I provide an annotated manuscript in which I point to several issues which are partly summarized above, plus some further issues (e.g., with terminology such as the use of the word "diabatic", "component of a scalar", etc.).

Reply: We appreciate the annotated manuscript and will consider the comments once we prepare the revised manuscript. The used of the word PVvert is not ideal. We will also ensure that conservative and non-conservative changes are clearly separated.