where ρ is the density of the medium and $\boldsymbol{\eta} = \nabla \times \boldsymbol{u} + 2\boldsymbol{\Omega}$ the vector of the absolute vorticity with the angular velocity of the Earth $\boldsymbol{\Omega}$. In the context of the primitive equations which are solved by the IFS this translates to

$$Q = \frac{1}{\rho}(f\boldsymbol{k} + \nabla \times \boldsymbol{u}_\mathrm{h}) \cdot \nabla\Theta, \tag{2}$$

where $f = 2\Omega\sin(\phi)$ is the Coriolis parameter, which represents the component of $\boldsymbol{\Omega}$ in $\boldsymbol{k}$ direction of the local rectangular coordinate system, and $\boldsymbol{u}_\mathrm{h}$ is the vector of the horizontal wind. The unit for the PV is the "potential vorticity unit" (pvu), with $1\,\mathrm{pvu} = 1 \times 10^{-6}\,\mathrm{m^2\,s^{-1}\,K\,kg^{-1}}$. The tropopause region is characterized by pronounced isentropic gradients of the potential vorticity, and the dynamic tropopause in the extratropics is commonly defined by a constant value of $Q = 2\,\mathrm{pvu}$ (Hoskins et al., 1985).

The vertical wind shear S^2 is calculated on half levels of the native vertical hybrid sigma-pressure level of the IFS, to retain a maximum amount of information in the gradient-based measure. The altitude at each model level is derived from the geopotential after vertically integrating the hydrostatic equation from the pressure and temperature profiles (for further information refer to the IFS documentation, ECMWF, 2016). The increasing vertical grid spacing with increasing altitude in the native coordinates results in a potential bias towards larger values of vertical wind shear at lower altitudes, which should be considered. However, the analysis focuses on the UTLS region, where the vertical grid spacing increases from about 300 m at 5 km altitude up to about 400 m at 20 km altitude (e.g., Hoffmann et al., 2019). Thus, the resolution bias should not have a large impact on the results presented. The central goal of this study is to quantify the occurrence of strong vertical wind shear S^2. For this, we define a threshold value $S_\mathrm{t}^2 = 4 \times 10^{-4}\,\mathrm{s^{-2}}$, marking the top end of the distribution of atmospheric vertical wind shear. The threshold value is selected based on the consideration that $S^2 \geq S_\mathrm{t}^2$ generally cannot be sustained by the average tropospheric static stability $\overline{N^2}_\mathrm{trop.}$, thus leading to low Richardson numbers and conditions favorable to turbulence. In contrast, average stratospheric static stability values of $\overline{N^2}_\mathrm{strat.} = 4 \times 10^{-4}\,\mathrm{s^{-2}}$ lead to Richardson numbers on the order of magnitude of $\mathcal{O}(1)$. Figure 1 illustrates the previous consideration, for one exemplary time step on which we will also focus in Sect. 3. The distribution of tropospheric static stability peaks around $N^2 = 1 \times 10^{-4}\,\mathrm{s^{-2}}$, with an overall average of $\overline{N^2}_\mathrm{trop.} = 1.33 \times 10^{-4}\,\mathrm{s^{-2}}$. The stratospheric static stability distribution amounts to an average value of $\overline{N^2}_\mathrm{strat.} = 5.20 \times 10^{-4}\,\mathrm{s^{-2}}$, which is shifted towards a larger value due to the above-average static stability which defines the TIL. The average stratospheric static stability above the TIL is generally closer to the value of $\overline{N^2}_\mathrm{strat.} = 4 \times 10^{-4}\,\mathrm{s^{-2}}$ (e.g., Birner et al., 2002). The choice of the threshold S_t^2 is furthermore motivated by previous research studies which indicate that the occurrence of strong vertical wind shear $S^2 \geq S_\mathrm{t}^2$ is largely restricted to tropopause altitudes (e.g., Dvoskin and Sissenwine, 1958; Kunkel et al., 2019).

3 Identification of a shear layer on 11 September 2017

We will start our analysis by initially focusing on a specific time step in September 2017. This study was performed in the context of the airborne research campaign WISE that took place during SON 2017 over the North Atlantic, which motivated the choice of the date. Our intention is to introduce our metrics first and then look at longer time periods afterwards. We start with the synoptic situation to provide the large-scale overview. Figure 2 shows a snapshot of the northern hemispheric potential temperature at LRT altitude on 11 September 2017, along with maxima of the horizontal wind at 200 hPa. The primary feature standing out is the tropopause break and the associated jet streaks of the horizontal wind. Over the Asian continent and reaching to the western Pacific, the tropopause break features a sharp meridional Θ(LRT) gradient with a single coherent STJ. Further west, the tropopause break is less sharp and features a characteristic sequence of Rossby wave patterns accompanied by individual jet streaks at varying latitudes.

The synoptic-scale wind systems are further illustrated in the vertical cross sections in Fig. 3. At 120° W (Fig. 3a), a pronounced polar jet maximum is visible at 50–60° N, which is also evident in the vertically integrated horizontal wind in Fig. 2. Further southward, the tropopause break at 30° N features comparatively weak westerly winds, followed by high-altitude easterlies in the tropics. The western North Atlantic (Fig. 3b) exhibits two distinct zonal west-wind maxima, which can be interpreted as the jet exit region of the STJ and the entry region of a jet streak of the PFJ. The cross section at 100° E (Fig. 3c) again shows a sequence of zonal winds, with high-altitude easterlies in the tropics, followed by the STJ, and further northward the rotational components of a cyclonic system over Siberia.

We proceed with the analysis of the vertical and geographical distribution of strong vertical wind shear exceeding the threshold value $S_\mathrm{t}^2 = 4 \times 10^{-4}\,\mathrm{s^{-2}}$. Figure 4 shows the zonally averaged relative occurrence frequency of strong vertical wind shear $S^2 \geq S_\mathrm{t}^2$ on a logarithmic color scale for the Northern Hemisphere on 11 September 2017. Figure 4a reveals three meridional regions of exceptional occurrence frequencies located in the UTLS region: one in the tropics, one at 30–40° N, and a third one at midlatitudes. Rearranging the grid volumes in a tropopause-relative vertical coordinate system depending on their vertical distance from the LRT (Fig. 4b and c) concentrates the maxima of strong wind shear occurrence in all three regions in a vertical layer of about 1–2 km in extent directly above the LRT. Secondary occurrence frequency maxima can be identified closely below the tropopause in all three meridional regions.