

We thank Dennis Hartmann for carefully reading our manuscript, and for his constructive comments. In the following we will respond to the various comments and point out any changes we intend to make to the paper based on them. Note that we have not provided exact manuscript corrections at this point, but we have provided the outline of planned changes. Line numbers and figure references in the reviewer's comments refer to the original manuscript. The reviewer's comments are in *black italics*; our responses are in [blue](#).

This paper is an interesting contribution to the literature on the impact of baroclinic shear on baroclinic lifecycles. Rather than using a single wavenumber to initialize the experiments, the authors add varying degrees of spatial white noise to the initial state. In cases of weak noise longer wavelengths grow via wave-wave interactions, (2,4,6) in the case of a base wavenumber of 6. These longer wavelengths are able to propagate toward the equator and lead to net poleward momentum flux in the case with large cyclonic barotropic shear (LC2) case as well as the LC1 case without the added cyclonic barotropic shear (LC1). If a high level of noise is added, shorter wavelengths, which are presumably more linearly unstable than wave 6, also develop early in the simulation and appear to break poleward in both the LC1 and LC2 cases. This leads to a situation where an initial stage of poleward wave breaking always occurs, but is always followed by equatorward wave propagation and breaking as the energy cascades to longer wavelengths that can propagate across the barotropic shear to the tropics. This leads one to conclude that equatorward wave propagation, poleward momentum flux, and poleward jet propagation must be a dominant feature of the general circulation, as is required by the global angular momentum balance

[Thank you for these encouraging summary remarks.](#)

Figure 3 , panels c and g are chosen at a particular time when wavenumber 4 dominates the image of PV. This misled me into thinking that wave 4 was growing by linear instability, which is not the thesis of the paper. Looking at Fig. 6 it is more obvious that this particular time is special. It would be good to note at this point that wave 2 is also evident in Fig. 3g or make some other comments to say that the dominance of wave 4 at this time is just transitory.

[Thank you for pointing this out. We indeed conclude that wave numbers 2 and 4 grow mostly due to non-linear interaction and not due to a simple linear instability. One argument here would also be that the initial state is much more unstable for other wavenumbers \(like 5 and 7\) than wavenumbers 4 and 2 \(see Fig. S1 in the supplement\).](#)

[Clarification in the text will be added. See also answers to comments on Fig. 3 and line 138 below.](#)

This is an interesting contribution and is fairly clearly written, with some exceptions that are noted below on a line-by-line and figure basis.

Comments on text:

Line 99: 'gradually'

[Will be done.](#)

115: Not sure what is meant by the initial phrase “Consistent with the energetics of the systems, “

We meant to express the consistency between the evolution of EKE and MKE with the PV dynamics during the additional noise-induced cycle. In the text we will adapt the sentence to be clearer on that.

117: Would a linear analysis of the zonal mean state at this time reveal that the most unstable wavenumber is 4? Is the energy of wave 4 coming from the mean state or WMF interactions?

This is a nice idea for potentially gaining deeper understanding of how wave 4 and 2 grow. However, we have conducted experiments on different levels of noisiness with resulting timings of secondary wave growth (comp. e.g. discussion of Fig. 5 in original manuscript). We found growth rates above the ones that wave 4 and 2 would follow if they would grow purely due to linear instability. Additionally such accelerated growth during different stages of the wave 6 cycle suggests that a sufficiently large amplitude of wave 6 is necessary for the growth of other waves independent of the current zonal mean state. Further, the applicability of linear stability theory is likely limited given the highly non-linear nature of the wave-breaking phase.

Fig. 3 in both cases, wavenumber 4 emerges as dominant around day 22-24. Why? It would be good at this point to say that you have picked out a particular time when wave 4 was dominant, and also point out that wavenumber 2 can also be seen at this time in panels C and G. The choice of time makes it look like it is mostly linear growth of wave 4, which is not consistent with the nonlinear theory that is actually the thesis of the paper. Clarification in the text will be added (planned to be inserted in line 117).

135: Is that because wavenumber 4 (and 2) can propagate toward the equator, while wavenumber 6 cannot in the LC2 state?

Thank you for this suggestion. Indeed, in our set up, wave numbers $k < 6$ seem to be able to propagate equatorwards more easily than $k = 6$ in the LC2 state. When wave 6 breaks, the LC2 state leads to a poleward wave-activity flux (comp. Fig. AC1 top right). However during the second wave breaking, where wave 4 and 2 dominate, the wave activity flux points towards the equator in both the LC1 and the LC2 setting. This can be also seen for strong noise in Fig. AC2 in both panels (LC1 and LC2) during the wave breaking of wave 4. Wave activity flux is equatorwards in both settings. We plan to include a comment on this.

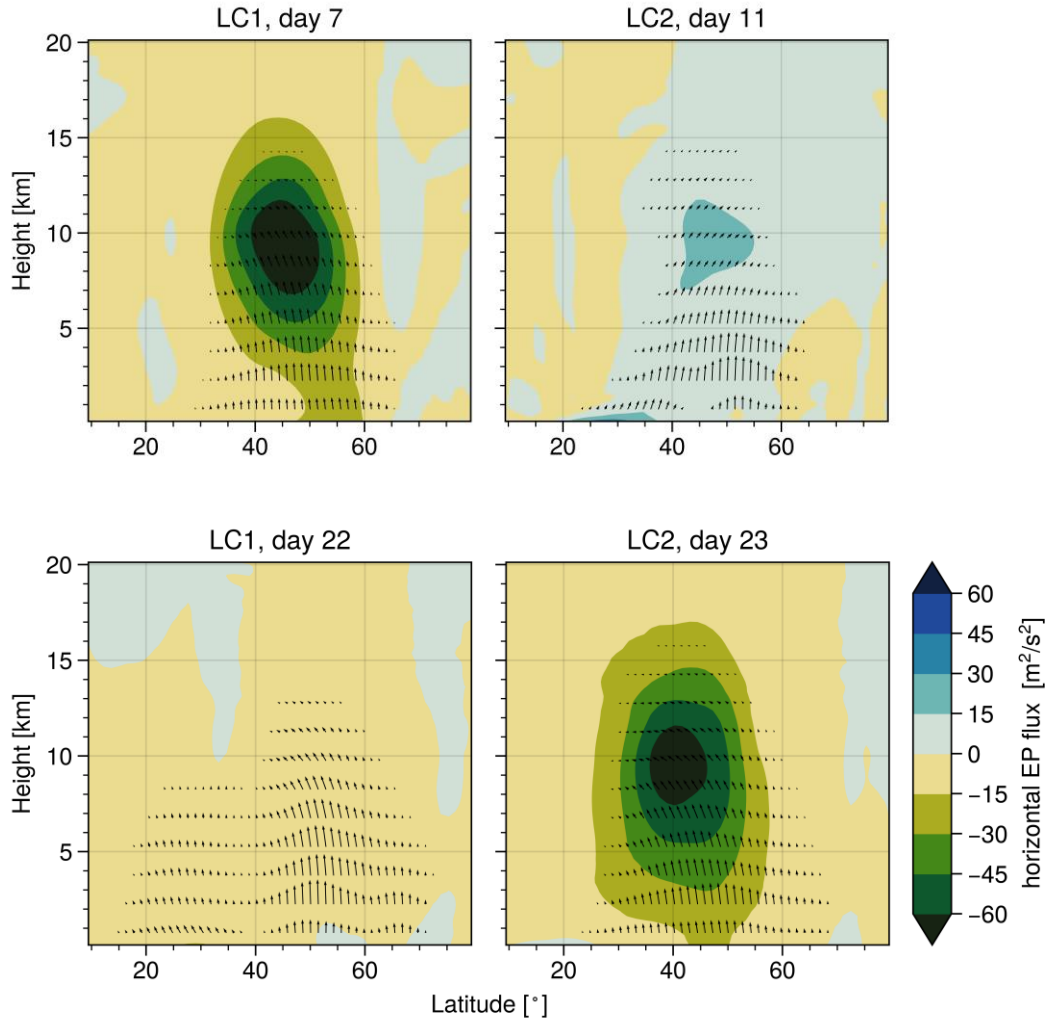


Fig. AC1: Eliassen-Palm flux for weak noise runs of LC1 (left) and LC2 (right) during the first (top) and the second wave breaking (bottom) indicated by arrows. Its horizontal component additionally is shown with the shading.

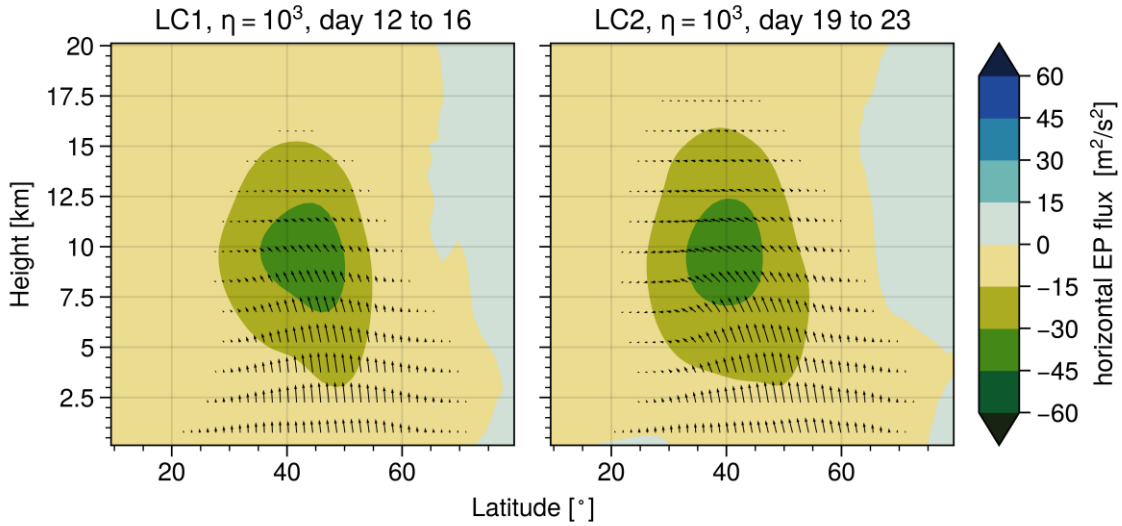


Fig. AC2: Eliassen-Palm flux indicated by arrows for strong noise runs of LC1 (left) and LC2 (right) during the five days around the EKE maximum, i.e. during the wave breaking of wave 4. Its horizontal component additionally is shown with the shading.

138: *On first reading, I did not quite get the physical reason for the emergence of wavenumber 4, which seems to be key. I don't see any reason for a state consisting of wavenumber 0 and 6 to create wavenumber 4 through nonlinear exchange, but if I look back at Fig. 3 panel G, I can see some wavenumber 2. It might help to point that out. Wavenumber 4 can propagate toward the equator and produce an LC1 outcome in the end.*

This will also be covered by the insertion in line 117 indicated above.

174: *If the wave breaking event creates a spectrum of wavenumbers, why is the initial noise so important to the evolution of the flow after the first wave-breaking phase?*

The non-linear triad interactions during the primary wave breaking event in monochromatically perturbed experiments only project on multiples of the perturbation wave number, i.e., in our case creates waves with wave numbers 6, 12, 18, 24, etc.. Some authors even limited their model to have a strict wave-6 symmetry (comp. Magnusdottir, G. and Haynes, 1996). Other wave numbers only become important because initialized as noise. Our understanding is that once wave 6 has become sufficiently strong, these wave numbers extract energy from wave 6 to grow faster than expected based on linear baroclinic instability theory.

Fig. 6 The legend "Specified wave 4" Is unclear. The other experiment was Specified wave 6, but it was allowed to evolve nonlinearly, whereas the curves for 4 and 3 seem to be extrapolations of their infinitesimal linear growth rates.

We will try to further clarify this in legend and caption. The dashed lines in Fig. 6 indicate an evolution with the linear growth rate of wave number k estimated via a linear fit of EKE during the initial growth phase of an experiment initialised with a single wave number k perturbation and no noise.

Fig. 6 If it is nonlinear wave exchange responsible for the growth of 2 and 4, why is their growth rate independent of the amplitude of wave 6? Their growth looks exponential, like they were linearly unstable.

Indeed, the growth of wave 4 and 2 is exponential, however with rates that seem to react to the amplitude of the driving wave 6. As mentioned in lines 168-171 in the original manuscript, the rates of $k=1,2,3,4,5$, which lie above the ones seen for linear instability in the reference runs, start to diverge in LC1 around day 8 and in LC2 around day 11. Selective triad interactions enhance the growth of waves 2 and 4 more than waves 1,3,5. In both experiments, this correlates with the peak in wave 6 EKE. Drops in growth rate correlate with a drop in wave 6 EKE. We interpret the growth of the noise, in particular wave 4 and 2, to be a combination of their own instability and accelerated growth via non-linear energy transfer.

194: *Did you mean to say, "In contrast to experiments with weak noise," As it is, it confused me. So in a case with white noise initialization, shorter wavelengths grow faster and tend to exhibit LC2 initial evolution, until the larger scales develop, which are able to propagate toward the equator, ending in a poleward jet shift and a more LC1-like final state.*

We agree with your comment. The wording of the sentence will be adapted.

265: *One might imagine a region of parameter space where the baroclinic growth of shorter wavelengths would be fast compared to the cascade to longer wavelengths in which the cyclonic state could be maintained by the poleward breaking of these shorter*

waves. It might also be possible that the shorter waves contribute their energy to a stationary wave, such as in the blocking ridge situation.

This is an intriguing idea. It would be very interesting for potential future work to explore this parameter space. In some way, the monochromatically and weakly perturbed LC2 experiments (Figs. 2 and 3) show this behaviour. There seems to be a threshold wave number k_0 above which we observe LC2 behaviour and below which we observe LC1 behaviour. We find high wave numbers to grow fastest in these experiments and lead to (quasi-)stable standing wave patterns (before short wave numbers start to dominate in cases with $\eta > 0$).

Clearly for the general circulation to work, the dominant direction of eddy propagation and breaking must be toward the equator to satisfy the angular momentum balance.

Thanks for stating clearly the consistency of our findings with this fundamental principle.

We agree that given that surface winds are easterly/westerly between low/high latitudes this is a nice heuristic argument for our core results. However, the dominance of equatorward breaking does not by itself preclude additional modifications due to poleward breaking, as long as the latter is weaker than the former (as is the case in the real atmosphere). The heuristic argument cannot answer the question of whether the total average is comprised of quasi-steady LC1 and LC2 states as described by Thorncroft et al. (1993) (with LC1 anomalies being stronger), or whether one of these exists as a purely transitory phenomenon (as implied by our results). We plan to include a related comment.